OLMSTED FALLS CITY SCHOOLS

MATHEMATICS COURSE OF STUDY

KINDERGARTEN THROUGH TWELFTH GRADE

May 2008

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STATEMENT OF APPROVAL

The Mathematics graded Course of Study was developed by a District Mathematics Committee. It was designed to implement the Ohio Department of Education Academic Content Standards for development of a graded course of study and will be articulated and implemented through in-service meetings to administrators and teachers.

This course of study will serve to stimulate further improvements of the Mathematics Program. All Olmsted Falls staff members are charged with the responsibility of using this course of study regularly and consistently in the development and presentation of the best instructional program possible for the Olmsted Falls students.

The K – 12 course of study will be approved by the Olmsted Falls board of Education and disseminated to all appropriate teachers and administrators for implementation.

RECOMMENDATION FOR PROGRAM ADOPTION

This Course of Study is recommended for approval by the following study committee members:

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This course of study was approved by the Olmsted Falls Board of Education at its meeting on _____May 15, 2008_____.

OLMSTED FALLS BOARD OF EDUCATION

PHILOSOPHY AND GOALS

The Board of Education of the Olmsted Falls School District believes that each student should participate in a school program that provides learning experiences, which are intended to result in intellectual and personal growth.

The Board of education believes that each student should be provided with these opportunities to further the development of:

1. Physical and mental health
2. Moral and ethical values
3. An appreciation of one's relation to the family, community, state, nation and world
4. Individual abilities and the use of them to reach the individual's fullest potential
5. Skills for participation in our democratic way of life
6. Skills of communication
7. Skills which will help in vocational endeavors
8. Scientific and aesthetic skills and appreciation
9. Constructive utilization of leisure time
10. A desire for continued self-advancement
11. A desire to excel
12. Curiosity

PROGRAM PHILOSOPHY and GOALS

The Olmsted Falls City School District believes that the mathematics program, based on state academic standards, ensures that all students have an opportunity to become mathematically literate, to become capable of extending their learning, to have an equal opportunity to learn, and to become informed citizens capable of understanding issues in a technological society. Knowledge of mathematics is an essential element in the development of the whole person.

Mathematics is more than a collection of isolated skills and concepts to be memorized and mastered. Mathematics is an integral link of 1) problem-solving, 2) reasoning and proving, 3) communicating mathematical thought, 4) representing ideas, and 5) making connections among mathematical concepts; therefore, an appropriate mathematics curriculum includes the investigation of the connections and the interplay among various mathematical topics and their applications at every grade level.

Whenever possible, students should have opportunities to learn mathematics through real-world contexts, including practical applications with real data and numbers often associated with situations and problems encountered in daily life. All students should be exposed to a mathematics program rich in technology including calculators, computers, and technology applications.

Further, students need to be able to understand and use mathematics effectively in everyday life and in a technological workplace. Mathematics literacy is needed to make everyday decisions such as choosing products to purchase, interpreting information in news reports, and selecting insurance or health plans. Mathematical thinking and problem solving are needed in the workplace, and those who understand and can use mathematics have significantly enhanced opportunities and options for a successful future. Mathematics plays a central role in modern culture, including aesthetic and recreational aspects, and an essential role in the scientific and technical community.

In keeping with the standards set forth by The National Council of Teachers of Mathematics (NCTM) and The Science and Mathematics Achievement Required for Tomorrow (SMART) consortium, we believe that all students should be provided access to the full range of mathematical topics. Since students' interests, goals, and achievements change as they mature and advance through high school, the mathematics program should be designed to work with students in order to provide viable options for them after graduation. Recognizing that individuals have different career objectives and may well pursue careers as yet undefined, we further recognize that **all students** have the right to learn significant mathematics and to develop power over mathematical ideas.

The goals of the mathematics program are that all students:

- Value mathematics
- Become confident in their ability to do mathematics
- Become mathematical problem-solvers
- Communicate mathematically
- Reason mathematically
- Apply mathematics to everyday life
- Utilize technology available for mathematical problem-solving

INTRODUCTION

The best possible resources on best practices in the field of Mathematics instruction were used and included in the Olmsted Falls Mathematics Course of Study K-12. These resources included: the National Council of Teachers Principles and Standards for School Mathematics, The Science and Mathematics Achievement Required for Tomorrow Consortium's (SMART) Mathematics Course of Study and as the core the Ohio Department of Education's Academic Content Standards for K-12 Mathematics

The ODE Mathematics Academic Content Standards were used as the fundamental core of this Course of Study. These standards have been designed to prepare all students for success in the workplace and post-secondary education. Competency in mathematics includes understanding mathematical concepts, facility with mathematical skills, and application of concepts and skills to problem-solving situations. These standards provide a comprehensive foundation for all students to think and reason mathematically and use mathematics knowledge and skills effectively.

The six standards that follow represent the mathematics content and processes all students should know and be able to use as they progress through school. These include:

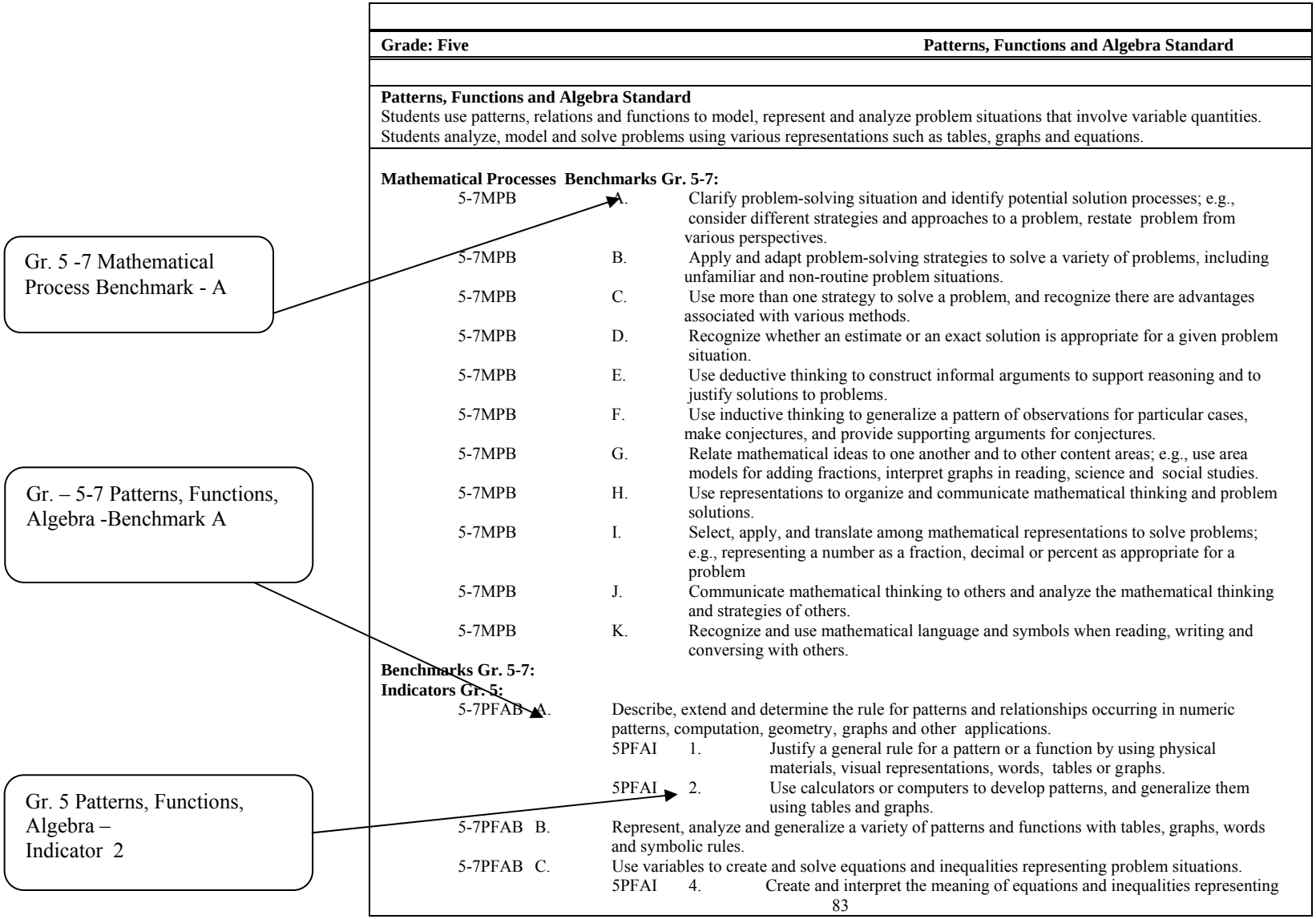
Content Standards:	Number, Number Sense and Operations
	Measurement
	Geometry and Spatial Sense
	Patterns, Functions, and Algebra
	Data Analysis and Probability
Process Standard:	Mathematical Processes

Throughout the five content standards, students will use mathematical processes, including reasoning, communication and representation skills, and appropriate technology within problem-solving situations. Making connections within mathematics and between mathematics and other disciplines is critical for student success in using mathematics effectively in school, work and daily life.

The Mathematics Course of Study has been organized into grade level bands that include standards, benchmarks, and grade-level indicators. The following defines these terms:

Standard:	An overarching goal or theme in mathematics. The standard statement describes, in broad terms, what students should know and be able to do as a result of the K-12 program
Benchmark:	A specific statement of what a student should know and be able to do at a specific time in his/her schooling. Benchmarks are used to measure a student's progress towards meeting the standard. Benchmarks are defined for grades 2, 4, 7, 10, and 12.
Grade-level Indicators:	A specific statement of the knowledge and/or skills that a student demonstrates at each grade level. These indicators serve as checkpoints that monitor progress toward the benchmarks.

The following are Keys to the interpretation of the grade level concept pages:
Excerpts from a sample page with keys:



ASSESSMENT

A strong, effective, aligned educational system has three parts. Standards are one important part. Curriculum and instruction is the second, and assessment aligned with the standards is the third part.

Ohio's comprehensive assessment system includes several types of assessment:

- Achievement tests
- Diagnostic tests
- Classroom assessments
- National and international assessments

Achievement Tests: provide the broadest picture of student performance. Ohio's achievement assessments, including the Ohio Achievement Test (OAT) and Ohio Graduation Test (OGT), are administered at specific grade levels and are based on the Ohio Academic Content Standards benchmarks. Statewide assessments measure student achievement and provide guidance for making program decisions and for decisions related to the allocation of resources at the state and local level.
Diagnostic Assessments: are administered annually at specific grade levels and are designed to give teachers and parents detailed information as to the strengths and weaknesses of individual students. They provide teachers with important information for instructional planning.
Classroom Assessments: on-going classroom assessments are one of the most important components in implementing an aligned standards-based system. Both informal and formal measures are to be used. Classroom assessments are to be aligned with what the students need to know and be able to do based on the content standards and then results are to be used to plan instruction that is tailored to the students' needs.

Samples of classroom assessment include: projects and investigations; portfolios, tests, quizzes, and short answer questions; extended response and essay questions; group tests; self-assessments, student reflection; teacher observations; short cycle assessments; formative assessments.

RESEARCH BASE: NUMBER AND OPERATIONS
GRADES K – 2

INTRODUCTION: The most fundamental concept in elementary school mathematics is that of number. Number sense gives students the confidence to solve problems and communicate ideas. Young children need opportunities to develop efficient strategies to compute fluently and to solve problems. In addition, students should have a variety of experiences investigating numbers in order to become numerically powerful. This power goes beyond the ability to compute; it encompasses an understanding of various meaning, relationships, properties, and procedures associated with numbers and operations.

RESEARCH BASE: "Counting in the absence of perceivable objects is the culmination of a rather intricate development of an ability to create unit items to be counted, first on the basis of conscious perception of external objects and then on the basis of internal representations." (Steff, 1994)
"Typical student beliefs about mathematical inquiry include the following: There is only one correct way to solve any mathematics problem; mathematics problems have only one correct answer; mathematics is done by individuals in isolation; mathematical problems can be solved quickly or not at all; mathematical problems and their solutions do not have to make sense; and that formal proof is irrelevant to processes of discovery and invention." (Schoenfeld, 1985, 1989a, 1989b)
"Developing fluency requires a balance and connection between conceptual understanding and computational proficiency. On the one hand, computational methods that are over-practiced without understanding are often forgotten or remembered incorrectly." (Hiebert, 1999; Kamii, Lewis, and Livingston, 1993; Hiebert and Linquist, 1990) "On the other hand, understanding without fluency can inhibit the problem-solving process." (Thornton, 1990)
"Research suggests using word problems as a basis for teaching addition and subtraction concepts, rather than teaching computational skills first and then applying them to solve problems." (Carpenter & Moser, 1983)

K – 2 Number and Number Sense

ESSENTIAL FOCUS:

1. Students should know how to count, order and compare objects and numbers. (K)
2. Students should know the concept of place value and the operations of addition and subtraction. (1st)
3. Students should know how to add and subtract fluently with single digit numbers. (2nd)
4. Students should know how to apply strategies for the addition and subtraction of two-digit numbers. (2nd)
5. Students should know it is possible to estimate quantities without knowing them exactly. (2nd)

RESEARCH BASE: MEASUREMENT
GRADES K – 2

INTRODUCTION: Measurement is an integral part of each of the strands of mathematics. By learning about how length, area, and volume are measured, students mentally structure and review their construction of space, both large-scale and small-scale. Measurement bridges two fundamental areas of school mathematics: geometry and number. A measure is the numerical value given to an attribute of an object. In primary grades, students need many experiences using nonstandard and standard forms of measurement.

RESEARCH BASE: Children's first understanding of length measure involves the direct comparison of objects. (Lindquist, 1989). Younger children often employ resemblance as the prime criteria for selecting a unit of area measure. Teaching experiments with area measure have revealed that second graders could develop a comprehensive understanding of area measure when they began by solving problems involving portioning and redistributing areas without measuring.

K-2 Measurement
ESSENTIAL FOCUS:

1. Students should know how to recognize intervals of time on a calendar (k).
2. Students should know how to use nonstandard units to measure (1^{st}).
3. Students should know how to use units of time (2^{nd}).
4. Students should know how to use standard units of measure (2^{nd}).

RESEARCH BASE: GEOMETRY
GRADES K – 2

INTRODUCTION: Proficiency in geometric reasoning develops in stages. These sequential states are associated with age. Children can be assisted by a progression of experiences that take them from recognizing shapes as wholes to recognizing explicit properties. Students need concrete experiences to develop spatial awareness and geometric knowledge. Students need to identify shapes in the world around them and to compare and sort these shapes according to their properties.

RESEARCH BASE: "Geometry is more than definitions; it is about describing relationships and reasoning. The notion of building understanding in geometry across the grades, from informal to more formal thinking, is consistent with the thinking of theorists and researchers." (Burger and Shaughnessy, 1986; Fuys, Geddes and Tischler, 1988; Senk, 1989; Van Hiele, 1986).

"Students advance through levels of thought in geometry. Van Hiele has characterized them as visual, descriptive, abstract/relational, and formal deduction. At the first level, students identify shapes and figures according to their concrete examples. At the next level students identify shapes according to their properties." (Van Hiele, 1986; Clements & Battista, 1992).

K-2 Geometry
ESSENTIAL FOCUS:

1. Students should know how to identify and describe common shapes (k).
2. Students should know how to identify and compare two-dimensional shapes (1^{st}).
3. Students should know how to manipulate shapes to recognize congruency with transformations. (2^{nd}).

RESEARCH BASE: PATTERNS, FUNCTIONS & ALGEBRA STRAND
GRADES K – 2

INTRODUCTION: Algebra is the fundamental language of mathematics. From the earliest grades of elementary school, students can begin to use simple algebraic thinking in their mathematical activities. They can observe that over time and across certain circumstances mathematical patterns occur. They can learn about functions by identifying and observing how changes in one variable may cause changes in other situations. A teacher's ability to help all students learn algebra depends in part on his or her awareness of the most important concepts and idea's: symbols, variable, structure, representation, patterns, graphing, expressions, equations, rules and functions.

RESEARCH BASE: "It is essential for students to learn algebra as a style of thinking involving the formalization of patterns, functions, and generalizations, as a set of competencies involving the presentation of quantitative relationships." (Silver, 1997)

"Two central themes of algebraic thinking are appropriate for young students. The first involves making generalizations and using symbols to represent mathematical ideas, and the second is representing and solving problems." (Carpenter and Levi, 1999)

K-2 Patterns, Functions & Algebra

ESSENTIAL FOCUS:

1. Students should know how to sort objects according to an attribute (K).
2. Students should know how to recognize and extend patterns (1^{st}).
3. Students should know how to generalize a pattern and determine a rule (2^{nd}).

RESEARCH BASE: DATA ANALYSIS AND PROBABILITY
GRADES K – 2

INTRODUCTION: Informal comparing, classifying, and counting activities can provide the mathematical beginnings for developing your learners' understanding of data analysis of data and statistics. Throughout the K – 2 years students should pose questions to investigate, organize the responses, and create representations of their data. Students should be encouraged to think clearly and check new ideas against what they already know. This will allow them to develop concept for making informed decisions. Ideas about probability in grades K – 2 should be informal and focus on judgments that children make through their experience. Activities that underlie experimental probability, such as tossing number cubes, should occur at this level, but the primary purpose for these activities is focused on other stands, such as numbers.

RESEARCH BASE: "The process or organizing and reducing data incorporates mental actions such as ordering, grouping and summarizing. The process of analyzing and interpreting data incorporates recognizing patterns and trends n data and making inferences and predictions from the data".

K-2 Data Analysis and Probability

ESSENTIAL FOCUS:

1. Students should know how to represent data by sorting and classifying objects according to attributes. (K)
1. Students should know how to collect, sort, and analyze data (1st).
2. Students should know how to read and interpret graphs and charts (2nd).
3. Students should know how to verify or justify their conclusions drawn from data (2nd).

Grade: Kindergarten — Number, Number Sense and Operations Standard

Number, Number Sense Standard:
Students demonstrate number sense including an understanding of number systems and operations, and how they relate to one another. Students compute fluently and make reasonable estimates using paper and pencil, technology-supported and mental methods.

Mathematical Processes Benchmarks K-2:

K-2MPB A. Use a variety of strategies to understand problem situations; e.g., discussing with peers, stating problems in own words, modeling problems with diagrams or physical materials, identifying a pattern.
K-2MPB B. Identify and restate in own words the question or problem and the information needed to solve the problem.
K-2MPB C. Generate alternative strategies to solve problems.
K-2MPB D. Evaluate the reasonableness of predictions, estimations and solutions.
K-2MPB E. Explain to others how a problem was solved.
K-2MPB F. Draw pictures and use physical models to represent problem situations and solutions.
K-2MPB G. Use invented and conventional symbols and common language to describe a problem situation and solution.
K-2MPB H. Recognize the mathematical meaning of common words and phrases, and relate everyday language to mathematical language and symbols.
K-2MPB I. Communicate mathematical thinking by using everyday language and appropriate mathematical language.

Benchmarks K-2:
Indicators Gr. K:

K-2NB A. Use place value concepts to represent whole numbers using numerals, words and physical models.
KNI 5. Relate, read and write numerals for single-digit numbers (0 to 9).

K-2NB B. Recognize, classify, compare and order whole numbers.
KNI 1. Compare and order whole numbers up to 10.
KNI 6. Construct multiple sets of objects each containing the same number of objects.
KNI 7. Compare the number of objects in two or more sets when one set has one or two more, or one or two fewer objects.
KNI 13. Recognize the number or quantity of sets up to 5 without counting; e.g., recognize without counting the dot arrangement on a domino as 5.

K-2NB C. Represent commonly used fractions using words and physical models

K-2NB D. Determine the value of a collection of coins and dollar bills.
KNI 9. Identify and state the value of a penny, nickel and dime.

K-2NB E. Make change using coins for values up to one dollar.

K-2NB F. Count, using numerals and ordinal numbers.
KNI 2. Explain rules of counting, such as each object should be counted once and that order does not change the number.

KNI 3. Count to twenty; e.g., in play situations or while reading number books.
KNI 4. Determine “how many” in sets (groups) of 10 or fewer objects.

K-2NB G. Model, represent and explain addition as combining sets and counting on.
KNI 8. Represent and use whole numbers in flexible ways, including relating, composing and decomposing numbers; e.g., 5 marbles can be 2 red and 3 green or 1 red and 4 green.

KNI 10. Model and represent addition as combining sets and counting on, and subtraction as take-away and comparison. For example:
a. Combine and separate small sets of objects in contextual situations; e.g., add or subtract one, two, or another small amount.
b. Count on (forward) and count back (backward) on a number line between 0 and 10.

K-2NB H. Model, represent and explain subtraction as comparison, take-away and part-to-whole.
KNI 8. Represent and use whole numbers in flexible ways, including relating, composing and decomposing numbers; e.g., 5 marbles can be 2 red and 3 green or 1 red and 4 green.
KNI 10. Model and represent addition as combining sets and counting on, and subtraction as take-away and comparison. For example:
a. Combine and separate small sets of objects in contextual situations; e.g., add or subtract one, two, or another small amount.
b. Count on (forward) and count back (backward) on a number line between 0 and 10.

K-2NB I. Model, represent and explain multiplication as repeated addition, rectangular arrays and skip counting.
KNI 6. Construct multiple sets of objects each containing the same number of objects.
KNI 7. Compare the number of objects in two or more sets when one set has one or two more, or one or two fewer objects.
KNI 11. Demonstrate joining multiple groups of objects, each containing the same number of objects; e.g., combining 3 bags of candy, each containing 2 pieces.

K-2NB J. Model, represent and explain division as sharing equally, repeated subtraction and rectangular arrays.
KNI 12. Partition or share a small set of objects into groups of equal size; e.g., sharing 6 stickers equally among 3 children.

K-2NB K. Demonstrate fluency in addition facts with addends through 9 and corresponding subtractions.
KNI 8. Represent and use whole numbers in flexible ways, including relating, composing and decomposing numbers; e.g., 5 marbles can be 2 red and 3 green or 1 red and 4 green.

K-2NB L. Demonstrate fluency in adding and subtracting multiples of 10, and recognize combinations that make 10.

K-2NB M. Add and subtract two-digit numbers with and without regrouping.

Grade: Kindergarten — Measurement Standard

Measurement Standard
Students estimate and measure to a required degree of accuracy and precision by selection and using appropriate units, tools and technologies.

Mathematical Processes Benchmarks K-2:

K-2MPB A. Use a variety of strategies to understand problem situations; e.g., discussing with peers, stating problems in own words, modeling problems with diagrams or physical materials, identifying a pattern.
K-2MPB B. Identify and restate in own words the question or problem and the information needed to solve the problem.
K-2MPB C. Generate alternative strategies to solve problems.
K-2MPB D. Evaluate the reasonableness of predictions, estimations and solutions.
K-2MPB E. Explain to others how a problem was solved.
K-2MPB F. Draw pictures and use physical models to represent problem situations and solutions.
K-2MPB G. Use invented and conventional symbols and common language to describe a problem situation and solution.
K-2MPB H. Recognize the mathematical meaning of common words and phrases, and relate everyday language to mathematical language and symbols.
K-2MPB I. Communicate mathematical thinking by using everyday language and appropriate mathematical language.

Benchmark K - 2:
Indicators Gr. K:

K-2MB A. Explain the need for standard units of measure.

K-2MB B. Select appropriate units for length, weight, volume (capacity) and time, using:
- objects; i.e., non-standard units;
- U.S. customary units; inch, foot, yard, ounce, pound, cup , quart, gallon, minute, hour, day, week and year;
- Metric units; centimeter, meter, gram and liter.

K-2MB C. Develop common referents for units of measure for length, weight, volume (capacity) and time to make comparisons and estimates.
KMI 1. Identify units of time (day, week, month, year) and compare calendar elements; e.g., weeks are longer than days.
KMI 2. Compare and order objects of different lengths, areas, weights and capacities; and use relative terms, such as longer, shorter, bigger, smaller, heavier, lighter, more and less.
KMI 4. Order events based on time. For example:
a. activities that take a long or short time;
b. review what we do first, next, last;
c. recall what we did or plan to do yesterday, today, tomorrow.

K-2MB D. Apply measurement techniques to measure length, weight and volume (capacity).
KMI 3. Measure length and volume (capacity) using uniform objects in the environment. For example, find:
a. how many paper clips long is a pencil;
b. how many small containers it takes to fill one big container using sand, rice, beans.

K-2MB E. Recognize that using different units of measurement will yield different numbers for the same measurement.

Grade: Kindergarten — Geometry and Spatial Sense Standard

Geometry and Spatial Sense Standard
Students identify, classify, compare and analyze characteristics, properties and relationships of one-, two- and three-dimensional geometric figures and objects. Students use spatial reasoning, properties of geometric objects, and transformations to analyze mathematical situations and solve problems.

Mathematical Processes Benchmarks K-2:

K-2MPB A. Use a variety of strategies to understand problem situations; e.g., discussing with peers, stating problems in own words, modeling problems with diagrams or physical materials, identifying a pattern.
K-2MPB B. Identify and restate in own words the question or problem and the information needed to solve the problem.
K-2MPB C. Generate alternative strategies to solve problems.
K-2MPB D. Evaluate the reasonableness of predictions, estimations and solutions.
K-2MPB E. Explain to others how a problem was solved.
K-2MPB F. Draw pictures and use physical models to represent problem situations and solutions.
K-2MPB G. Use invented and conventional symbols and common language to describe a problem situation and solution.
K-2MPB H. Recognize the mathematical meaning of common words and phrases, and relate everyday language to mathematical language and symbols.
K-2MPB I. Communicate mathematical thinking by using everyday language and appropriate mathematical language.

Benchmark K - 2:
Indicators Gr. K:

K-2GSB A. Describe and create plane figures: circle, rectangle, square, triangle, hexagon, trapezoid, parallelogram and rhombus, and identify them in the environment.

K-2GSB B. Describe solid objects: cube, rectangular prism, sphere, cylinder, cone and pyramid, and identify them in the environment.

K-2GSB C. Sort and compare two-dimensional figures and three-dimensional objects according to their characteristics and properties.
KGSI 1. Identify and sort two-dimensional shapes and three-dimensional objects. For example:
a. Identify and describe two-dimensional figures and three-dimensional objects from the environment using the child's own vocabulary.
b. Sort shapes and objects into groups based on student-defined categories.
c. Select all shapes or objects of one type from a group.
d. Build two-dimensional figures using paper shapes or tangrams; build simple three-dimensional objects using blocks.

K-2GSB D. Identify, explain and model (superposition, copying) the concept of shapes being congruent and similar.

K-2GSB E. Recognize two- and three-dimensional objects from different positions.

K-2GSB F. Describe location, using comparative (before, after), directional (above, below), and positional (first, last) words.
KGSI 2. Name and demonstrate the relative position of objects as follows:
a. place objects over, under, inside, outside, on, beside, between, above, below, on top of, upside-down, behind, in back of, in front of;

b. describe placement of objects with terms, such as on, inside, outside, above, below, over, under, beside, between, in front of, behind.

K-2GSB G. Identify and draw figures with line symmetry.

Grade: Kindergarten — Patterns, Functions and Algebra Standard

Patterns, Functions and Algebra Standard
Students use patterns, relations and functions to model, represent and analyze problem situations that involve variable quantities. Students analyze, model and solve problems using various representations such as tables, graphs and equations.

Mathematical Processes Benchmarks K-2:

K-2MPB A. Use a variety of strategies to understand problem situations; e.g., discussing with peers, stating problems in own words, modeling problems with diagrams or physical materials, identifying a pattern.
K-2MPB B. Identify and restate in own words the question or problem and the information needed to solve the problem.
K-2MPB C. Generate alternative strategies to solve problems.
K-2MPB D. Evaluate the reasonableness of predictions, estimations and solutions.
K-2MPB E. Explain to others how a problem was solved.
K-2MPB F. Draw pictures and use physical models to represent problem situations and solutions.
K-2MPB G. Use invented and conventional symbols and common language to describe a problem situation and solution.
K-2MPB H. Recognize the mathematical meaning of common words and phrases, and relate everyday language to mathematical language and symbols.
K-2MPB I. Communicate mathematical thinking by using everyday language and appropriate mathematical language.

Benchmarks K-2:
Indicators Gr. K:

K-2PFAB A. Sort, classify and order objects by size, number and other properties, and describe the attributes used.
KPFAI 1. Sort, classify and order objects by size, number and other properties. – Such as shape, color, weight and physical properties**.** For example:
a. Identify how objects are alike and different.
b. Order three events or objects according to a given attribute, such as time or size.
c. Recognize and explain how objects can be classified in more than one-way.
d. Identify what attribute was used to sort groups of objects that have already been sorted.

K-2PFAB B. Extend sequences of sounds and shapes or simple number patterns, and create and record similar patterns.
KPFAI 2. Identify, create, extend, draw and copy sequences of sounds (such as musical notes), shapes (such as buttons, leaves or blocks), motions (such as hops or skips), and numbers from 1 to 10.

K-2PFAB C. Create and extend patterns, and describe the rule in words.
KPFAI 3. Describe orally the pattern of a given sequence.

K-2PFAB D. Model problem situations, using objects, pictures, numbers and other symbols.
KPFAI 4. Model a problem situation using physical materials.

K-2PFAB E. Solve open sentences and explain strategies.

K-2PFAB F. Represent an unknown quantity as a variable using a symbol, such as □, ○, △.

K-2PFAB G. Describe and compare qualitative and quantitative changes.

Grade: Kindergarten — Data Analysis and Probability Standard

Data Analysis and Probability Standard:
Students pose questions and collect, organize, represent, interpret and analyze data to answer those questions. Students develop and evaluate inferences, predictions and arguments that are based on data.

Mathematical Processes Benchmarks K-2:

K-2MPB A. Use a variety of strategies to understand problem situations; e.g., discussing with peers, stating problems in own words, modeling problems with diagrams or physical materials, identifying a pattern.
K-2MPB B. Identify and restate in own words the question or problem and the information needed to solve the problem.
K-2MPB C. Generate alternative strategies to solve problems.
K-2MPB D. Evaluate the reasonableness of predictions, estimations and solutions.
K-2MPB E. Explain to others how a problem was solved.
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K-2MPB G. Use invented and conventional symbols and common language to describe a problem situation and solution.
K-2MPB H. Recognize the mathematical meaning of common words and phrases, and relate everyday language to mathematical language and symbols.
K-2MPB I. Communicate mathematical thinking by using everyday language and appropriate mathematical language.

Benchmark K-2:
Indicators Gr. K:

K-2DPB A. Pose questions and gather data about everyday situations and familiar objects.
KDPI 1. Gather and sort data in response to questions posed by teacher and students; e.g., how many sisters and brothers?, what color shoes?

K-2DPB B. Sort and classify objects by attributes, and organize data into categories in a simple table or chart.
KDPI 2. Arrange objects in a floor or table graph according to attributes, such as use, size, color or shape.
KDPI 3. Select the category or categories that have the most or fewest objects in a floor or table graph.

K-2DPB C. Represent data using objects, picture graphs and bar graphs.

K-2DPB D. Describe the probability of chance events as more, less or equally likely to occur.

Grade: One — Number, Number Sense and Operations Standard

Number, Number Sense Standard:
Students demonstrate number sense including an understanding of number systems and operations, and how they relate to one another. Students compute fluently and make reasonable estimates using paper and pencil, technology-supported and mental methods.

Mathematical Processes Benchmarks K-2:

K-2MPB A. Use a variety of strategies to understand problem situations; e.g., discussing with peers, stating problems in own words, modeling problems with diagrams or physical materials, identifying a pattern.
K-2MPB B. Identify and restate in own words the question or problem and the information needed to solve the problem.
K-2MPB C. Generate alternative strategies to solve problems.
K-2MPB D. Evaluate the reasonableness of predictions, estimations and solutions.
K-2MPB E. Explain to others how a problem was solved.
K-2MPB F. Draw pictures and use physical models to represent problem situations and solutions.
K-2MPB G. Use invented and conventional symbols and common language to describe a problem situation and solution.
K-2MPB H. Recognize the mathematical meaning of common words and phrases, and relate everyday language to mathematical language and symbols.
K-2MPB I. Communicate mathematical thinking by using everyday language and appropriate mathematical language.

Benchmarks K - 2:
Indicators Gr. 1:

K-2NB A. Use place value concepts to represent whole numbers using numerals, words and physical models.
- 1NI 3. Read and write the numerals for numbers to 100
- 1NI 5. Use place value concepts to represent whole numbers using numerals, words, expanded notation, and physical models with ones and tens. For example:
  a. develop a system to group and count by twos, fives and tens.
  b. Identify patterns and groupings in a 100's chart and relate to place value concepts.
  c. Recognize the first digit of a two digit number as the most important to indicate size of a number and the nearness to 10 or 100.

K-2NB B. Recognize, classify, compare and order whole numbers.
- 1NI 1. Use ordinal numbers to order objects; e.g., first, second third.
- 1NI 2. Recognize and generate equivalent forms for the same number using physical models, words and number expressions; e.g., concept of ten is described by "10 blocks", or full tens frame, or numeral 10, or 5 + 5, or 15 – 5, or one less than 11, or my brother's age.
- 1NI 4. Count forward to 100, count backwards from 100, and count forward or backward starting at any number between 1 and 100.
- 1NI 15. Demonstrate that equal means "the same as" using visual representations.

K-2NB C. Represent commonly used fractions using words and physical models.
- 1NI 9. Represent commonly used fractions using words and physical models for halves, thirds and fourths, recognizing fractions are represented by equal size parts of a whole and of a set of objects.

K-2NB D. Determine the value of a collection of coins and dollar bills.

1NI 6. Identify and state the value of a penny, nickel, dime, quarter and dollar.

1NI 7. Determine the value of a small collection of coins (with a total value up to one dollar) using 1 or 2 different type coins, including pennies, nickels, dimes and quarters.

K-2NB E. Make change using coins for values up to one dollar.

1NI 8. Show different combinations of coins that have the same value.

K-2NB F. Count, using numerals and ordinal numbers.

1NI 4. Count forward to 100, count backwards from 100, and count forward or backward starting at any number between 1 and 100.

K-2NB G. Model, represent and explain addition as combining sets and counting on.

1NI 10. Model, represent and explain addition as combining sets (part + part = whole) and counting on. For example:

a. Model and explain addition using physical materials in contextual situations.

b. Draw pictures to model addition.

c. Write number sentences to represent addition.

d. Explain that adding two whole numbers yields a larger whole number.

K-2NB H. Model, represent and explain subtraction as comparison, take-away and part-to-whole.

1NI 11. Model, represent and explain subtraction as take-away and comparison. For example:

a. Model and explain subtraction using physical materials in contextual situations.

b. Draw pictures to model subtraction.

c. Write number sentences to represent subtraction.

d. Explain that subtraction of whole numbers yields an answer smaller than the original number.

1NI 12. Use conventional symbols to represent the operations of addition and subtraction.

K-2NB I. Model, represent and explain multiplication as repeated addition, rectangular arrays and skip counting.

1NI 13. Model and represent multiplication as repeated addition and rectangular arrays in contextual situations; e.g., four people will be at my party and if I want to give 3 balloons to each person, how many balloons will I need to buy?

K-2NB J. Model, represent and explain division as sharing equally, repeated subtraction and rectangular arrays.

1NI 14. Model and represent division as sharing equally in contextual situations; e.g., sharing cookies.

K-2NB K. Demonstrate fluency in addition facts with addends through 9 and corresponding subtractions.

1NI 16. Develop strategies for basic addition facts, such as:

a. counting all;

b. counting on;

c. one more, two more;

d. doubles;

e. doubles plus or minus one;

f. make ten;

g. using tens frames;
h. identity property (adding zero).

1NI 17. Develop strategies for basic subtraction facts, such as:
a. relating to addition (for example, think of 7 - 3 = ? as “3 plus ? equals 7”);
b. one less, two less;
c. all but one (for example, 8 - 7, or 5 - 4);
d. using tens frames;
e. missing addends.

K-2NB L. Demonstrate fluency in adding and subtracting multiples of 10, and recognize combinations that make 10.

1NI 16. Develop strategies for basic addition facts, such as:
a. counting all;
b. counting on;
c. one more, two more;
d. doubles;
e. doubles plus or minus one;
f. make ten;
g. using tens frames;
h. identity property (adding zero).

1NI 17. Develop strategies for basic subtraction facts, such as:
a. relating to addition (for example, think of 7 - 3 = ? as “3 plus ? equals 7”);
b. one less, two less;
c. all but one (for example, 8 - 7, or 5 - 4);
d. using tens frames;
e. missing addends.

K-2NB M. Add and subtract two-digit numbers with and without regrouping.

Measurement Standard
Students estimate and measure to a required degree of accuracy and precision by selection and using appropriate units, tools and technologies.

Mathematical Processes Benchmarks K-2

K-2MPB A. Use a variety of strategies to understand problem situations; e.g., discussing with peers, stating problems in own words, modeling problems with diagrams or physical materials, identifying a pattern.
K-2MPB B. Identify and restate in own words the question or problem and the information needed to solve the problem.
K-2MPB C. Generate alternative strategies to solve problems.
K-2MPB D. Evaluate the reasonableness of predictions, estimations and solutions.
K-2MPB E. Explain to others how a problem was solved.
K-2MPB F. Draw pictures and use physical models to represent problem situations and solutions.
K-2MPB G. Use invented and conventional symbols and common language to describe a problem situation and solution.
K-2MPB H. Recognize the mathematical meaning of common words and phrases, and relate everyday language to mathematical language and symbols.
K-2MPB I. Communicate mathematical thinking by using everyday language and appropriate mathematical language.

Benchmark K - 2:
Indicators Gr. 1:

K-2MB A. Explain the need for standard units of measure.
1 MBI 1. Recognize and explain the need for fixed units and tools for measuring length and weight; e.g., rulers and balance scales.

K-2MB B. Select appropriate units for length, weight, volume (capacity) and time, using:
- objects; i.e., non-standard units;
- U.S. customary units; inch, foot, yard, ounce, pound, cup , quart, gallon, minute, hour, day, week and year;
- Metric units; centimeter, meter, gram and liter.

K-2MB C. Develop common referents for units of measure for length, weight, volume (capacity) and time to make comparisons and estimates.
1 MBI 2. Tell time to the hour and half hour on digital and analog (dial) timepieces.
1 MBI 3. Order a sequence of events with respect to time; e.g., summer, fall, winter and spring; morning, afternoon and night.

K-2MB D. Apply measurement techniques to measure length, weight and volume (capacity).
1 MBI 4. Estimate and measure weight using non-standard units; e.g., blocks of uniform size.
1 MBI 5. Estimate and measure lengths using non-standard and standard units; i.e., centimeters, inches and feet.

K-2MB E. Recognize that using different units of measurement will yield different numbers for the same measurement.

Grade: One — Geometry and Spatial Sense Standard

Geometry and Spatial Sense Standard
Students identify, classify, compare and analyze characteristics, properties and relationships of one-, two- and three-dimensional geometric figures and objects. Students use spatial reasoning, properties of geometric objects, and transformations to analyze mathematical situations and solve problems.

Mathematical Processes Benchmarks K-2

K-2MPB A. Use a variety of strategies to understand problem situations; e.g., discussing with peers, stating problems in own words, modeling problems with diagrams or physical materials, identifying a pattern.
K-2MPB B. Identify and restate in own words the question or problem and the information needed to solve the problem.
K-2MPB C. Generate alternative strategies to solve problems.
K-2MPB D. Evaluate the reasonableness of predictions, estimations and solutions.
K-2MPB E. Explain to others how a problem was solved.
K-2MPB F. Draw pictures and use physical models to represent problem situations and solutions.
K-2MPB G. Use invented and conventional symbols and common language to describe a problem situation and solution.
K-2MPB H. Recognize the mathematical meaning of common words and phrases, and relate everyday language to mathematical language and symbols.
K-2MPB I. Communicate mathematical thinking by using everyday language and appropriate mathematical language.

Benchmarks K - 2:
Indicators Gr. 1:

K-2GSB A. Describe and create plane figures: circle, rectangle, square, triangle, hexagon, trapezoid, parallelogram and rhombus, and identify them in the environment.
1GSI 2. Create new shapes by combining or cutting apart existing shapes.

K-2GSB B. Describe solid objects: cube, rectangular prism, sphere, cylinder, cone and pyramid, and identify them in the environment.
1GSI 3. Identify the shapes of the faces of three-dimensional objects.

K-2GSB C. Sort and compare two-dimensional figures and three-dimensional objects according to their characteristics and properties.
1GSI 1. Identify, compare and sort two-dimensional shapes; eg., square, circle, ellipse, triangle, rectangle, rhombus, trapezoid, parallelogram, pentagon and hexagon. For example:
a. Recognize and identify triangles and rhombuses independent of position, shape or size;
b. Describe two-dimensional shapes using attributes such as number of sides and number of vertices (corners or angles).

K-2GSB D. Identify, explain and model (superposition, copying) the concept of shapes being congruent and similar.

K-2GSB E. Recognize two- and three-dimensional objects from different positions.

K-2GSB F. Describe location, using comparative (before, after), directional (above, below), and positional (first, last) words.
1GSI 4. Extend the use of location words to include distance (near, far, close to) and directional words (left, right).

K-2GSB G. Identify and draw figures with line symmetry.

1GSI 5. Copy figures and draw simple two-dimensional shapes from memory.

Grade: One — Patterns, Functions and Algebra Standard

Patterns, Functions and Algebra Standard:
Students use patterns, relations and functions to model, represent and analyze problem situations that involve variable quantities. Students analyze, model and solve problems using various representations such as tables, graphs and equations.

Mathematical Processes Benchmarks K-2

K-2MPB A. Use a variety of strategies to understand problem situations; e.g., discussing with peers, stating problems in own words, modeling problems with diagrams or physical materials, identifying a pattern.
K-2MPB B. Identify and restate in own words the question or problem and the information needed to solve the problem.
K-2MPB C. Generate alternative strategies to solve problems.
K-2MPB D. Evaluate the reasonableness of predictions, estimations and solutions.
K-2MPB E. Explain to others how a problem was solved.
K-2MPB F. Draw pictures and use physical models to represent problem situations and solutions.
K-2MPB G. Use invented and conventional symbols and common language to describe a problem situation and solution.
K-2MPB H. Recognize the mathematical meaning of common words and phrases, and relate everyday language to mathematical language and symbols.
K-2MPB I. Communicate mathematical thinking by using everyday language and appropriate mathematical language.

Benchmarks K -2:
Indicators Gr. 1:

K-2PFAB A. Sort, classify and order objects by size, number and other properties, and describe the attributes used.
1PFAI 1. Sort, classify and order objects by two or more attributes, such as color and shape, and explain how objects were sorted.

K-2PFAB B. Extend sequences of sounds and shapes or simple number patterns, and create and record similar patterns.
1PFAI 2. Extend sequences of sounds, shapes or simple number patterns, and create and record similar patterns. For example:
a. Analyze and describe patterns with multiple attributes using numbers and shapes; e.g., AA, B, aa, b, AA, B, aa, b,…
b. Continue repeating and growing patterns with materials, pictures and geometric items; e.g., XO, XOO, XOOO, XOOOO.

K-2PFAB C. Create and extend patterns, and describe the rule in words.
1PFAI 3. Describe orally the basic unit or general plan of a repeating or growing pattern.

K-2PFAB D. Model problem situations, using objects, pictures, numbers and other symbols.
1PFAI 5. Describe orally and model a problem situation using words, objects or number phrase or sentence. (e.g.,– if, then, since, because, not, all, some….) .

K-2PFAB E. Solve open sentences and explain strategies.
1PFAI 4. Solve open sentences by representing an expression in more than one way using the commutative property; e.g., 4 + 5 = 5 + 4 or the number of blue balls plus red balls is the same as the number of red balls plus blue balls (R + B = B + R).

K-2PFAB F. Represent an unknown quantity as a variable using a symbol, such as □, ○, △.

K-2PFAB G. Describe and compare qualitative and quantitative changes.

Grade: One — Data Analysis and Probability Standard

Data Analysis and Probability Standard:
Students pose questions and collect, organize, represent, interpret and analyze data to answer those questions. Students develop and evaluate inferences, predictions and arguments that are based on data.

Mathematical Processes Benchmarks K-2

K-2MPB A. Use a variety of strategies to understand problem situations; e.g., discussing with peers, stating problems in own words, modeling problems with diagrams or physical materials, identifying a pattern.
K-2MPB B. Identify and restate in own words the question or problem and the information needed to solve the problem.
K-2MPB C. Generate alternative strategies to solve problems.
K-2MPB D. Evaluate the reasonableness of predictions, estimations and solutions.
K-2MPB E. Explain to others how a problem was solved.
K-2MPB F. Draw pictures and use physical models to represent problem situations and solutions.
K-2MPB G. Use invented and conventional symbols and common language to describe a problem situation and solution.
K-2MPB H. Recognize the mathematical meaning of common words and phrases, and relate everyday language to mathematical language and symbols.
K-2MPB I. Communicate mathematical thinking by using everyday language and appropriate mathematical language.

Benchmarks K -2:
Indicators Gr. 1:

K-2DPB A. Pose questions and gather data about everyday situations and familiar objects.
1DPI 5. Construct a question that can be answered by using information from a graph.

K-2DPB B. Sort and classify objects by attributes, and organize data into categories in a simple table or chart.
1DPI 1. Identify multiple categories for sorting data.
1DPI 2. Collect and organize data into charts using tally marks.
1DPI 6. Arrange five objects by an attribute, such as size or weight, and identify the ordinal position of each object
1DPI 7. Answer questions about the number of objects represented in a picture graph, bar graph or table graph; e.g., category with most, how many more in a category compared to another, how many altogether in two categories.

K-2DPB C. Represent data using objects, picture graphs and bar graphs.
1DPI 3. Display data in picture graphs with units of 1 and bar graphs with intervals of 1.
1DPI 4. Read and interpret charts, picture graphs and bar graphs as sources of information to identify main ideas, draw conclusions, and make predictions.

K-2DPB D. Describe the probability of chance events as more, less or equally likely to occur.
1DPI 8. Describe the likelihood of simple events as possible/impossible and more likely/less likely; e.g., when using spinners or number cubes in classroom activities.

Grade: Two — Number, Number Sense and Operations Standard

Number, Number Sense Standard:
Students demonstrate number sense including an understanding of number systems and operations, and how they relate to one another. Students compute fluently and make reasonable estimates using paper and pencil, technology-supported and mental methods.

Mathematical Processes Benchmark K-2:

K-2MPB A. Use a variety of strategies to understand problem situations; e.g., discussing with peers, stating problems in own words, modeling problems with diagrams or physical materials, identifying a pattern.
K-2MPB B. Identify and restate in own words the question or problem and the information needed to solve the problem.
K-2MPB C. Generate alternative strategies to solve problems.
K-2MPB D. Evaluate the reasonableness of predictions, estimations and solutions.
K-2MPB E. Explain to others how a problem was solved.
K-2MPB F. Draw pictures and use physical models to represent problem situations and solutions.
K-2MPB G. Use invented and conventional symbols and common language to describe a problem situation and solution.
K-2MPB H. Recognize the mathematical meaning of common words and phrases, and relate everyday language to mathematical language and symbols.
K-2MPB I. Communicate mathematical thinking by using everyday language and appropriate mathematical language.

Benchmarks Gr. K - 2:
Indicators Gr. 2:

K-2NB A. Use place value concepts to represent whole numbers using numerals, words and physicals models.

K-2NB B. Recognize, classify, compare and order whole numbers.

2NI 1. Use place value concepts to represent, compare and order whole numbers using physical models, numerals and words, with ones, tens and hundreds. For example:
a. Recognize 10 can mean "10 ones" or a single entity (1 ten) through physical models and trading games.
b. Read and write 3-digit numerals (e.g., 243 as two hundred forty-three, 24 tens and 3 ones, or 2 hundreds and 43 ones, etc.) and construct models to represent each.

2NI 2. Recognize and classify numbers as even or odd.

K-2NB C. Represent commonly used fractions using words and physical models.

2NI 5. Represent fractions (halves, thirds, fourths, sixths and eighths), using words, numerals and physical models. For example:
a. Recognize that a fractional part can mean different amounts depending on the original quantity.
b. Recognize that a fractional part of a rectangle does not have to be shaded with contiguous parts.
c. Identify and illustrate parts of a whole and parts of sets of objects.
d. Compare and order physical models of halves, thirds and fourths in relations to 0 and 1.

K-2NB D. Determine the value of a collection of coins and dollar bills.

2NI 4. Represent and write the value of money using the ¢ sign and in decimal form when using the $ sign.

K-2NB E. Make change using coins for values up to one dollar.
2NI 3. Count money and make change using coins and a dollar bill.

K-2NB F. Count, using numerals and ordinal numbers.

K-2NB G. Model, represent and explain addition as combining sets and counting on.

K-2NB H. Model, represent and explain subtraction as comparison, take-away and part-to-whole.
2NI 6. Model, represent and explain subtraction as comparison, take-away and part-to-whole; e.g., solve missing addend problems by counting up or subtracting, such as "I had six baseball cards, my sister gave me more, and I now have ten. How many did she give me?" can be represented as 6 + ? = 10 or 10 - 6 = ?

K-2NB I. Model, represent and explain multiplication as repeated addition, rectangular arrays and skip counting.
2NI 7. Model, represent and explain multiplication as repeated addition, rectangular arrays and skip counting.

K-2NB J. Model, represent and explain division as sharing equally, repeated subtraction and rectangular arrays.
2NI 8. Model, represent and explain division as sharing equally and repeated subtraction.

K-2NB K. Demonstrate fluency in addition facts with addends through 9 and corresponding subtractions.
2NI 10. Demonstrate fluency in addition facts with addends through 9 and corresponding subtractions; e.g., 9 + 9 = 18, or 18 – 9 = 9.

K-2NB L. Demonstrate fluency in adding and subtracting multiples of 10, and recognize combinations that make 10.
2NI 11. Add and subtract multiples of 10.

K-2NB M. Add and subtract two-digit numbers with and without regrouping.
2NI 9. Model and use the commutative property for addition.
2NI 12. Demonstrate multiple strategies for adding and subtracting 2 or 3 digit whole numbers, such as:
a. Compatible numbers;
b. Compensatory numbers;
c. Informal use of commutative and associative properties of addition;
d. Using a calculator;
e. Regrouping.
2NI 13. Estimate the results of whole number addition and subtraction problems using front-end estimation, and judge the reasonableness of the answers.

Grade: Two — Measurement Standard

Measurement Standard
Students estimate and measure to a required degree of accuracy and precision by selecting and using appropriate units, tools and technologies.

Mathematical Processes Benchmark K-2:

K-2MPB A. Use a variety of strategies to understand problem situations; e.g., discussing with peers, stating problems in own words, modeling problems with diagrams or physical materials, identifying a pattern.
K-2MPB B. Identify and restate in own words the question or problem and the information needed to solve the problem.
K-2MPB C. Generate alternative strategies to solve problems.
K-2MPB D. Evaluate the reasonableness of predictions, estimations and solutions.
K-2MPB E. Explain to others how a problem was solved.
K-2MPB F. Draw pictures and use physical models to represent problem situations and solutions.
K-2MPB G. Use invented and conventional symbols and common language to describe a problem situation and solution.
K-2MPB H. Recognize the mathematical meaning of common words and phrases, and relate everyday language to mathematical language and symbols.
K-2MPB I. Communicate mathematical thinking by using everyday language and appropriate mathematical language.

Benchmarks K – 2:
Indicators Gr. 2:

K-2MB A. Explain the need for standard units of measure.

K-2MB B. Select appropriate units for length, weight, volume (capacity) and time, using:
- Objects; i.e., non-standard units;
- U.S. customary units: inch, foot, yard, ounce, pound, cup, quart, gallon, minute, hour, day, week and year;
- Metric units: centimeter, meter, gram and liter

2MI 1. Identify and select appropriate units of measure for:
a. length – centimeters, meters, inches, feet or yards;
b. volume (capacity) – liters, cups, pints or quarts;
c. weight – grams, ounces or pounds;
d. time – hours, half-hours, quarter-hours or minutes and time designations, a.m. or p.m.

K-2MB C. Develop common referents for units of measure for length, weight, volume (capacity) and time to make comparisons and estimates.
2MI 2. Establish personal or common referents for units of measure to make estimates and comparisons; e.g., the width of a finger is a centimeter, a large bottle of soda pop is 2 liters, a small paper clip weighs about one gram.
2MI 4. Tell time to the nearest minute interval on digital and to the nearest 5 minute interval on analog (dial) timepieces.

K-2MB D Apply measurement techniques to measure length, weight and volume (capacity).
2MI 5. Estimate and measure the length and weight of common objects, using metric and U.S. customary units, accurate to the nearest unit.
2MI 6. Select and use appropriate measurement tools; e.g., a ruler to draw a segment 3 inches long, a measuring cup to place 2 cups of rice in a bowl, a scale to weigh 50 grams of candy.

K-2MB E. Recognize that using different units of measurement will yield different numbers for the same measurement.

2MI 3. Describe and compare the relationships among units of measure, such as centimeters and meters; inches, feet and yards; cups, pints and quarts; ounces and pounds; and hours, half-hours, and quarter-hours; e.g., How many inches in a foot?

2MI 7. Make and test predictions about measurements, using different units to measure the same length or volume.

Grade: Two — Geometry and Spatial Sense Standard

Geometry and Spatial Sense Standard
Students identify, classify, compare and analyze characteristics, properties and relationships of one-, two- and three-dimensional geometric **figures and objects. Students use spatial reasoning, properties of geometric objects, and transformations to analyze mathematical situations and solve problems.**

Mathematical Processes Benchmark K-2:

K-2MPB A. Use a variety of strategies to understand problem situations; e.g., discussing with peers, stating problems in own words, modeling problems with diagrams or physical materials, identifying a pattern.
K-2MPB B. Identify and restate in own words the question or problem and the information needed to solve the problem.
K-2MPB C. Generate alternative strategies to solve problems.
K-2MPB D. Evaluate the reasonableness of predictions, estimations and solutions.
K-2MPB E. Explain to others how a problem was solved.
K-2MPB F. Draw pictures and use physical models to represent problem situations and solutions.
K-2MPB G. Use invented and conventional symbols and common language to describe a problem situation and solution.
K-2MPB H. Recognize the mathematical meaning of common words and phrases, and relate everyday language to mathematical language and symbols.
K-2MPB I. Communicate mathematical thinking by using everyday language and appropriate mathematical language.

Benchmarks K -2:
Indicators Gr. 2:

K-2GSB A. Describe and create plane figures: circle, rectangle, square, triangle, hexagon, trapezoid, parallelogram and rhombus, and identify them in the environment
2GSI 1. Identify, describe, compare and sort three-dimensional objects (eg., cubes, spheres, prisms, cones, cylinders and pyramids) according to the shape of the faces or the numbers of faces, edges or vertices.
2GSI 2. Predict what new shapes will be formed by combining or cutting apart existing shapes.

K-2GSB B. Describe solid objects: cube, rectangular prism, sphere, cylinder, cone and pyramid, and identify them in the environment.

K-2GSB C. Sort and compare two-dimensional figures and three-dimensional objects according to their characteristics and properties.
2GSI 1. Identify, describe, compare and sort three-dimensional objects (eg., cubes, spheres, prisms, cones, cylinders and pyramids) according to the shape of the faces or the numbers of faces, edges or vertices.

K-2GSB D. Identify, explain and model (superposition, copying) the congruent and similar.
2GSI 4. Identify and determine whether two-dimensional shapes are congruent (same shape and size) or similar (same shape, different size) by copying or using superposition (lay one thing on top of another).

K-2GSB E. Recognize two- and three-dimensional objects from different positions.
2GSI 3. Recognize two-dimensional shapes and three-dimensional objects from different positions.

K- 2GSB F. Describe location, using comparative (before, after), directional (above, below), and positional (first, last) words.

K-2GSB G. Identify and draw figures with line symmetry.

2GSI 5. Create and identify two-dimensional figures with a line of symmetry; e.g., What letter shapes, logos, polygons are symmetrical?

Grade: Two — Patterns, Functions and Algebra Standard

Patterns, Functions and Algebra Standard
Students use patterns, relations and functions to model, represent and analyze problem situations that involve variable quantities. Students analyze, model and solve problems using various representations such as tables, graphs and equations.

Mathematical Processes Benchmark K-2:

K-2MPB A. Use a variety of strategies to understand problem situations; e.g., discussing with peers, stating problems in own words, modeling problems with diagrams or physical materials, identifying a pattern.
K-2MPB B. Identify and restate in own words the question or problem and the information needed to solve the problem.
K-2MPB C. Generate alternative strategies to solve problems.
K-2MPB D. Evaluate the reasonableness of predictions, estimations and solutions.
K-2MPB E. Explain to others how a problem was solved.
K-2MPB F. Draw pictures and use physical models to represent problem situations and solutions.
K-2MPB G. Use invented and conventional symbols and common language to describe a problem situation and solution.
K-2MPB H. Recognize the mathematical meaning of common words and phrases, and relate everyday language to mathematical language and symbols.
K-2MPB I. Communicate mathematical thinking by using everyday language and appropriate mathematical language.

Benchmarks K - 2:
Indicators Gr. 2:

K-2PFAB A. Sort, classify and order objects by size, number and other properties, and describe the attributes used.

K-2PFAB B. Extend sequences of sounds and shapes or simple number patterns, and create and record similar patterns.
2PFAI 1. Extend simple number patterns (both repeating and growing patterns), and create similar patterns using different objects, such as using physical materials or shapes to represent numerical patterns.

K-2PFAB C. Create and extend patterns, and describe the rule in words.
2PFAI 2. Use patterns to make generalizations and predictions; e.g., determine a missing element in a pattern.
2PFAI 3. Create new patterns with consistent rules or plans, and describe the rule or general plan of existing patterns.

K-2PFAB D. Model problem situations, using objects, pictures, numbers and other symbols.
2PFAI 4. Use objects, pictures, numbers and other symbols to represent a problem situation.

K-2PFAB E. Solve open sentences and explain strategies.
2PFAI 5. Understand equivalence and extend the concept to situations involving symbols; e.g., $4 + 5 = 9$ and $9 = 4 + 5$, and $4 + 5 = 3 + 6 =$ △ + □.

K-2PFAB F. Represent an unknown quantity as a variable using a symbol, such as □, ○, △.

2PFAI 6. Use symbols to represent unknown quantities and identify values for symbols in an expression or equation using addition and subtraction; e.g., □ + ○ = 10, △ -2 = 4.

K-2PFAB G. Describe and compare qualitative and quantitative changes.

2PFAI 7. Describe qualitative and quantitative changes, especially those involving addition and subtraction; e.g., a student growing taller versus a student growing two inches in one year.

Grade: Two — Data Analysis and Probability Standard

Data Analysis and Probability Standard:
Students pose questions and collect, organize, represent, interpret and analyze data to answer those questions. Students develop and evaluate inferences, predictions and arguments that are based on data.

Mathematical Processes Benchmark K-2:

- K-2MPB A. Use a variety of strategies to understand problem situations; e.g., discussing with peers, stating problems in own words, modeling problems with diagrams or physical materials, identifying a pattern.
- K-2MPB B. Identify and restate in own words the question or problem and the information needed to solve the problem.
- K-2MPB C. Generate alternative strategies to solve problems.
- K-2MPB D. Evaluate the reasonableness of predictions, estimations and solutions.
- K-2MPB E. Explain to others how a problem was solved.
- K-2MPB F. Draw pictures and use physical models to represent problem situations and solutions.
- K-2MPB G. Use invented and conventional symbols and common language to describe a problem situation and solution.
- K-2MPB H. Recognize the mathematical meaning of common words and phrases, and relate everyday language to mathematical language and symbols.
- K-2MPB I. Communicate mathematical thinking by using everyday language and appropriate mathematical language.

Benchmarks K - 2:
Indicators Gr. 2:

K-2DPB A. Pose questions and gather data about everyday situations and familiar objects.
- 2DPI 6. Recognize that data may vary from one population to another; e.g., favorite TV shows of students and of parents.

K-2DPB B. Sort and classify objects by attributes, and organize data into categories in a simple table or chart.
- 2DPI 1. Pose questions, use observations, interviews and surveys to collect data, and organize data in charts, picture graphs and bar graphs.
- 2DPI 4. Write a few sentences to describe and compare categories of data represented in a chart or graph, and make statements about the data as a whole.

K-2DPB C. Represent data using objects, picture graphs and bar graphs.
- 2DPI 2. Read, interpret and make comparisons and predictions from data represented in charts, line plots, picture graphs and bar graphs.
- 2DPI 3. Read and construct simple timelines to sequence events.
- 2DPI 5. Identify untrue or inappropriate statements about a given set of data.

K-2DPB D. Describe the probability of chance events as more, less or equally likely to occur.
- 2DPI 7. List some of the possible outcomes of a simple experiment, and predict whether given outcomes are more, less or equally likely to occur.
- 2DPI 8. Use physical models and pictures to represent possible arrangements of 2 or 3 objects.

RESEARCH BASE: NUMBERS AND OPERATIONS
GRADES 3 – 4

INTRODUCTION:

Number sense gives students confidence to use mathematics in everyday life. In grades 3 – 4, students' understanding of the base-ten number system is extended to larger numbers and to decimals. Using benchmark values, common fractions are compared to each other and to whole numbers. Computational fluency is essential and may be accomplished using various methods. The focus at this level is multiplication and division. This fluency should be developed with an understanding of arithmetic operations and problem solving. Estimation is encouraged to judge the reasonableness of an answer. A range of strategies should be employed and students should be able to explain their thinking. When appropriate, calculators and computers can enhance and extend mathematical understanding at this level.

RESEARCH BASE:

Elementary and middle school students may have limited ability with place value (Sowder, 1992a). Sowder reports that middle school students are able to identify the place values of the digits that appear in a number, but they cannot use the knowledge confidently in context (for example, students have trouble determining how many boxes of 100 candy bars could be packed from 48,638 candy bars).
Students make a variety of errors in multi-digit addition and subtraction calculations (Brown & van Lehn, 1982). Given traditional instruction, a substantial number of 4^{th} graders are not able to subtract some whole numbers successfully. (Fuson, 1992). Student errors suggest students interpret and treat multi-digit numbers as single-digit numbers placed adjacent to each other, rather than using place-value meanings for the digits in different positions. (Fuson, 1992). With specially designed instructions 2^{nd} graders are able to understand place value and to add and subtract four-digit numbers more accurately and meaningfully than 3^{rd} graders receiving traditional instruction (Fuson, 1992). Research also suggests students interpret multiplication of whole numbers mainly as repeated addition. This interpretation is inadequate for many multiplication problems and can lead to restrictive intuitive notions such as 'multiplication always makes larger' (Greer, 1992).

Elementary and middle school students make several errors when they operate on decimals and fractions (Benander & Clement, 1985; Kouba et al., 1988; Peck & Jencks, 1981; Wearne & Hiebert, 1988). For example, many middle-school students cannot add 4 + 0.3 correctly or 7 1/6 + 3 ½ (Kouba et al., 1988; Wearne & Hiebert, 1988). These errors are due in part to the fact that students lack essential concepts about decimals and fractions and have memorized procedures that they apply incorrectly. Interventions to improve concept knowledge can lead to increased ability by 5^{th} graders to add and subtract decimals correctly (Wearne & Hiebert, 1988).

Students of all ages misunderstand multiplication and division (Bell et al., 1984; Graeber & Tirosh, 1988; Greer, 1992). Commonly held misconceptions include 'multiplication always make larger,' 'division always makes smaller,' 'the divisor must always be smaller than the dividend.' Students may correctly select multiplication as the operation needed to calculate the cost of gasoline when the amount and unit cost are integers, then select division for the same problem when the amount and unit cost are decimal numbers (Bell et al., 1981).
Upper elementary and middle school students often do not realize that decimal fractions represent concrete objects that can be measured by units, tenths of units, hundredths of units, and so on (Hiebert, 1992). For example, students have trouble writing decimals for shaded parts of rectangular regions divided into 10 or 100 equal parts (Hiebert & Wearne, 1986). Other students have little understanding of the value represented by each of the digits of a decimal number of how the value of the number is the sum of the value of its digits. Students of all ages have problems choosing the largest or smallest in a set of decimals with different numbers of digits to the right of the decimal points (Carpenter et al., 1981; Hiebert & Wearne, 1986; Resnick et al., 1989). Upper elementary school students can establish rich meanings for decimal symbols and do a variety of decimal tasks well after specially designed instruction using base-10 blocks (Wearne & Hiebert, 1988, 1989).

Instruction that focuses on the meaning of fractions and decimals forms a basis on which to build a good understanding of the relationship between fractions and decimals. Instruction that merely shows how to translate between the two forms does not provide a conceptual base for understanding the relationship (Markowits & Sowder, 1991).
Lower elementary students do not have procedures to compare the size of whole numbers. By 4th grade, students generally have no difficulty comparing the sizes of whole numbers up to four digits (Sowder, 1992). Students are less successful when the number of digits is much larger or when more than two numbers are to be compared. This might be due to increased memory requirements of working with more or larger numbers (Sowder, 1988). Upper elementary and middle school students taught traditionally cannot successfully compare decimal numbers (Sowder, 1988, 1992). Rather they over generalize the features of the whole number system to the decimal numbers (Resnick et al., 1989). They apply a 'more digits make bigger' rule (According to which .1814- > .385). After specially designed instruction, which develops good meanings for decimal symbols, many students are able to compare decimal numbers with understanding by 5th grade (Wearne & Hiebert, 1988). Upper elementary and middle school students taught traditionally cannot compare fractions successfully (Sowder, 198). Students' difficulties here indicate they treat the numerator and the denominator separately. Specially designed instruction to teach meanings for fractions can help to improve ordering fractions by as early as the end of the 5th grade (Behr et al., 1984).

The use of calculators in K – 12 mathematics does not hinder the development of basic computation skills and frequently improves concept development and paper-and-pencil skills, both in basics operations and in problem solving (Hembree & Dessart, 1986; Kaput, 1992). The use of calculators in testing produces higher scores than paper-and-pencil efforts in problem solving as well as in basic operations (Hembree & Dessart, 1986).
Middle school and even high school students may have limited understanding about the nature and purpose of estimation. They often think it is inferior to exact computation and equate it with guessing (Sowder, 1992b), so that they do not believe estimation is useful (Sowder & Wheeler, 1989). Students who see estimation as a valuable tactic for obtaining information use estimation more frequently and successfully (Threadgil-Sowder, 1984).
Good estimators use a variety of estimating tactics and switch easily between them. They have a good understanding of place value and the meaning of operations, and they are skilled in mental computation. Poor estimators rely on algorithms that are more likely to yield the exact answer. They lack an understanding of the notion and value of estimation and often describe it as 'guessing' (Sowder, 1992b). Before 6th grade, students develop very few estimation skills from the natural experiences (Case & Sowder, 1990; Sowder, 1992b). As a result, some researchers caution that teaching estimation to young children may have as its single effect that they master specific procedures in a superficial manner (Sowder, 1992b).

3 – 4 Number and Number Sense

ESSENTIAL FOCUS:

1. Students should know how to add and subtract fluently with two- and three-digit numbers (3rd).
2. Students should know how to model, represent and explain multiplicative reasoning (3rd).
3. Students should know factions are parts of a whole (3rd).
4. Student should know how to multiply fluently (4th).
5. Students should know how to model, represent and explain division (4th)
6. Students should know when answers are reasonable. (4th).

RESEARCH BASE: MEASUREMENT
GRADES 3 - 4

INTRODUCTION: Measurement is a line that connects ideas within areas of mathematics and bridges mathematics to other disciplines. Using measurement, students in grades 3 – 5 explore questions related to their environment. They investigate real-world situations involving measurement of temperature, perimeter, angles, area, and volume. Students should select appropriate tools and units of measurement and recognize factors that affect precision. In addition, it is important that students realize that all measurements are approximations.

RESEARCH BASE: In order to realize that arbitrary measures are not reliable, a child must reconcile the varying lengths and numbers of arbitrary units and reason transitively. On the other hand to use a standard device such as a 30 cm or meter ruler to make pair by pair direct comparisons of lengths of objects is a less demanding task. It also has the advantage that it appears to be a real-world meaningful activity. (Boulton-Lewis et al., 1994, pg. 130).

It was concluded…. The use of drawing in the development of area concepts helps children to develop abstractions and to recognize the units that go to make up a shape. (Wheatley and Reynold, 1996).

"Most researchers agree that there are three components of measuring: conservation, transitivity, and units and unit iteration: (Chapin & Johnson, p. 177). "Students in the United states must become proficient in using both the English systems and the metric system of measurement" (Chapin & Johnson, p. 195).

"Tools …. help children reason about the mathematically important components of activity so that invariants like unit are represented physically and then mentally" (Lehrer & Schauble, p. 282). "Students find it very difficult to decompose and then recompose shapes or even to see one shape as a composition of others, an idea that is fundamental to conservation: (Lehrer, Jenkins, and Osana, p. 88).

3-4 Measurement
ESSENTIAL FOCUS:

1. Students should know tht standard units of measurements are necessary (3^{rd}).
2. Students should know how to convert whole number measurements within each system (4^{th}).
3. Students should know how to define and determine perimeter (4^{th}).

RESEARCH BASE: GEOMETRY
GRADES 3 - 4

INTRODUCTION: As students investigate the attributes and properties of geometric shapes, they develop more precise descriptions of the relationships they discover. They are learning to reason and make, test, and justify conjectures about these relationships. Students need to extend geometric knowledge and develop spatial reasoning ability by visualizing geometric relationships. Students need to extend geometric knowledge and develop spatial reasoning ability by visualizing geometric relationships. Spatial understanding is necessary for interpreting, understanding and appreciating our inherently geometric world.

RESEARCH BASE: Students advance through levels of thought in geometry. Van Hiele has characterized them as visual, descriptive, abstract, relational, and formal deduction (Van Hiele, 1986; Clements & Battista, 1992). At the first level, students identify shapes and figures according to their concrete examples For example, a student may say that a figure is a rectangle because it looks like a door. At the second level, students identify shape according to their properties, and here a student might think of a rhombus as a figure with four equal sides. At the third level, students can identify relationships between classes of figures (e.g., a square is a rectangle) and can discover properties of classes of figures by simple logical deduction.

Progress from one of Van Hieles' levels to the next is more dependent upon instruction than age. Given traditional instruction, middle-school students perform at levels one or two (Clements & Battista, 1992). Some evidence suggests it is possible for students to understand the abstract properties of geometric figures by 5th grade (Clements & Battista, 1989, 1990, 1992; Wirszup, 1976).

With well-designed activities, appropriate tools and teachers' support, students can make and explore conjectures about geometry and can learn to reason carefully about geometric ideas from the earliest years of schooling. Geometry is more than definitions; it is about describing relationships and reasoning. The notion of building understanding in geometry across the grades, from informal to more formal thinking, is consistent with the thinking of theorists and researchers (Burger and Shaughnessy, 1986; Fuys, Geddes, and Tischler, 1988; Send, 1989, Van Hiele, 1986).

3-4 Geometry
ESSENTIAL FOCUS:

1. Students should know how to identify, compare, and analyze attributes of two- and three-dimensional shapes (3rd).
2. Students should know how to use appropriate vocabulary to describe geometric properties and relationships (4th).

RESEARCH BASE: PATTERNS, FUNCTIONS & ALGEBRA STRAND
GRADES 3 - 4

INTRODUCTION: Algebra is a style of thinking where students study patterns and relationships and learn to use them in daily life. Patterns are the basis for reasoning about regularity and consistency. As students move into upper elementary, they need to generalize these patterns and express the relationships using language, symbols, tables and graphs.

Change is an important mathematical idea that can be studied using the tools of algebra. Research indicates that this is not an area that students typically understand with much depth. Using graphs and tables, students in grades 3 – 4 start to notice and describe change. As they look at sequences, they can begin to distinguish between arithmetic growth and geometric growth.

RESEARCH BASE: Preliminary research hints that students have difficulty making connections between mathematical expressions, sentences, and sequences that share common structural patterns. They focus instead upon incidental similarities or differences (Ericksen, 1991).

Students of all ages often do not view the quality sign of equations as a symbol or the equivalence between the left and the right side of the equation, but rather interpret it as a sign to begin calculating (Kieran, 1992) Students who are encouraged initially to use trial-and-error substitution develop a better notion of the equivalence of the two sides of the equation and are more successful in applying more formal methods later on (Kieran, 1988, 1989).

3-4 Patterns, Functions and Algebra

ESSENTIAL FOCUS:

1. Students should know how to write a mathematical expression or equation using symbols and letters (3rd).
2. Students should know how to apply the distributive property (4th).

RESEARCH BASE: DATA ANALYSIS AND PROBABILITY
GRADES 3 - 4

INTRODUCTION: The analysis of data helps students begin to understand the world around them. Books, newspapers, the Internet, and other media are filled with graphical displays. With such widespread use, data analysis becomes very critical. Hence, it is important that students in grades 3 – 5 progress from reading data to interpreting tables and graphs.

Moreover, students should formulate questions to investigate relevant issues in their lives. Further more, they must develop the skills of collecting valid data, organizing it, describing its central tendency and variability, and creating meaningful representations that can be used to make predictions and inferences.

Students at this level will also begin to investigate the concepts of probability. Through experiments, students will explore the frequency of various outcomes and use the results to make predictions.

RESEARCH BASE: Research suggests that a good notion of representativeness may be a prerequisite to grasping the definitions for measures of location like mean, median, or mode. Students can acquire notions of representativeness after they state seeing data sets as entities to be described and summarized rather than as "unconnected" individual values. This occurs typically around the 4th grade (Mokros & Russell, 1992).

Research suggests students should be introduced first to location measures that connect with their emerging concept of the "middle" such as the median, and later in the middle-school grades, to the mean. Premature introduction into the algorithm for computing the mean divorced from a meaningful context may block students from understanding what averages are for (Mokros & Russell, 1992; Pollatsek et al, 1981).

The concept of the mean is quite difficult for students of all ages to understand even after several years of formal instruction. Several difficulties have been documented in the literature: Students of all ages can talk about the algorithm for computing the mean and relate it to limited contexts, but cannot use it meaningfully in problems (Makros & Russell, 1992; Pollatsek, Lime and Well, 1981); upper elementary- and middle-school students believe that the mean of a particular data set is not one precise numerical value but an approximation that can have one of several values (Makros & Russell, 1992).

Research presents somewhat contradictory results on elementary children's understanding of probability. Piagetian research says lower elementary children have no conception of probability (Piaget & Inhelder, 1975; Shayer & Adey, 1981), but other studies indicate that even lower elementary-school children have probabilistic intuitions upon which probability instruction can build. Falk et al. (1980) presented elementary-school students with two sets, each containing blue and yellow elements. Each time, one color was pointed out as the payoff color. The students had to choose the set from which they would draw at random a "payoff element" to be rewarded. From the age of six, children began to select the more probable set systematically. The ability to choose correctly precedes the ability to explain these choices.

Upper elementary students can give correct examples for certain, possible, and impossible events, but cannot calculate probability of independent and dependent events even after instruction on the procedure (Fischbein & Gazit, 1983) that this is partly because students at this age tend to create "part to part" rather than "part to whole" comparisons (e.g., 9 men and 11 women rather than 15% of men and 10% of women).

Extensive research points to several misconceptions about probabilistic reasoning that are similar at all age levels and are found even among experienced researchers (Kahneman, Slovic & Tversky, 1982; Shaughnessy, 1992). One common misconception is the idea of reprsentativeness, according to which an event is believed to be probable to the extent that it is "typical". For example, many people believe that after a run of heads in coin tossing, tails should be more likely to come up. Another common error is estimating the likelihood of events based on how easily instances of it can be brought to mind.

3-4 Data Analysis and Probability

ESSENTIAL FOCUS:

1. Students should know how to translate information among charts, tables, and graphs (3^{rd}).
2. Students should know how to describe the likelihood of event (3^{rd}).
3. Students should know how to appropriately represent and interpret data (4^{th}).
4. Students should know how to relate basic probability concepts to realistic situations (4^{th}).

Grade: Three — Number, Number Sense and Operations Standard

Number, Number Sense Standard:
Students demonstrate number sense including an understanding of number systems and operations, and how they relate to one another. Students compute fluently and make reasonable estimates using paper and pencil, technology-supported and mental methods.

Mathematical Processes Benchmarks Gr. 3 - 4:

- 3-4MPB A. Apply and justify the use of a variety of problem-solving strategies; e.g., make an organized list, guess and check.
- 3-4MPB B. Use an organized approach and appropriate strategies to solve multi-step problems.
- 3-4MPB C. Interpret results in the context of the problem being solved; e.g., the solution must be a whole number of buses when determining the number of buses necessary to transport students.
- 3-4MPB D. Use mathematical strategies to solve problems that relate to other curriculum areas and the real world; e.g., use a timeline to sequence events; use symmetry in artwork.
- 3-4MPB E. Link concepts to procedures and to symbolic notation; e.g., model 3 x 4 with a geometric array, represent one-third by dividing an object into three equal parts.
- 3-4MPB F. Recognize relationships among different topics within mathematics; e.g., the length of an object can be represented by a number.
- 3-4MPB G. Use reasoning skills to determine and explain the reasonableness of a solution with respect to the problem situation.
- 3-4MPB H. Recognize basic valid and invalid arguments, and use examples and counter examples, models, number relationships, and logic to support or refute.
- 3-4MPB I. Represent problem situations in a variety of forms (physical model, diagram, in words or symbols), and recognize when some ways of representing a problem may be more helpful than others.
- 3-4MPB J. Read, interpret, discuss and write about mathematical ideas and concepts using both everyday and mathematical language.
- 3-4MPB K. Use mathematical language to explain and justify mathematical ideas, strategies and solutions.

Benchmarks Gr. 3 - 4:
Indicators Gr. 3:

- 3-4NB A. Use place value structure of the base-ten number system to read, write, represent and compare whole numbers and decimals.
  - 3NI 1. Identify and generate equivalent forms of whole numbers; e.g., 36, 30 + 6, 9 x 4, 46 - 10, number of inches in a yard.
  - 3NI 2. Use place value concepts to represent whole numbers and decimals using numerals, words, expanded notation and physical models. For example:
    - a. Recognize 100 means "10 tens" as well as a single entity (1 hundred) through physical models and trading games.
    - b. Describe the multiplicative nature of the number system; e.g., the structure of 3205 as 3 x 1000 plus 2 x 100 plus 5 x 1.
    - c. Model the size of 1000 in multiple ways; e.g., packaging 1000 objects into 10 boxes of 100, modeling a meter with centimeter and decimeter strips, or gathering 1000 pop-can tabs.
    - d. Explain the concept of tenths and hundredths using physical models, such as metric pieces, base ten blocks, decimal squares or money.
  - 3NI 3. Use mathematical language and symbols to compare and order; e.g., less than, greater than, at most, at least, $<$, $>$, $=$, $\leq$, $\geq$.

3-4NB B. Recognize and generate equivalent representations for whole numbers, fractions and decimals.

3-4NB C. Represent commonly used fractions and mixed numbers using words and physical models.
- 3NI 5. Represent fractions and mixed numbers using words, numerals and physical models.

3-4NB D. Use models, points of reference and equivalent forms of commonly used fractions to judge the size of fractions and to compare, describe and order them.
- 3NI 3. Use mathematical language and symbols to compare and order; e.g., less than, greater than, at most, at least, <, >, =, ≤, ≥.
- 3NI 6. Compare and order commonly used fractions and mixed numbers using number lines, models (such as fraction circles or bars), points of reference materials in contextual situations. (Such as more or less than ½) and equivalent forms found using physical or visual models.
- 3NI 7. Recognize and use decimal and fraction concepts and notations as related ways of representing parts of a whole or a set; e.g., 3 of 10 marbles are red can also be described as 3/10 and 3 tenths are red.

3-4NB F. Count money and make change using both coins and paper bills.
- 3NI 4. Count money and make change using coins and paper bills to ten dollars.

3-4NB G. Model and use commutative and associative properties for addition and multiplication.
- 3NI 11. Model and use the commutative and associative properties for addition and multiplication.

3-4NB H. Use relationships between operations, such as subtraction as the inverse of addition and division as the inverse of multiplication.
- 3NI 10. Explain and use relationships between operations, such as:
  a. relate addition and subtraction as inverse operations;
  b. relate multiplication and division as inverse operations;
  c. relate addition to multiplication (repeated addition);
  d. relate subtraction to division (repeated subtraction).Multiply and divide 2- and 3-digit numbers by a single-digit number, without remainders for division. [NB-K] Demonstrate fluency in multiplication facts through 10 and corresponding division facts
- 3NI 11. Model and use the commutative and associative properties for addition and multiplication.
- 3NI 12. Add and subtract whole numbers with and without regrouping.
- 3NI 13. Demonstrate fluency in multiplication facts through 10 and corresponding division facts.
- 3NI 14. Multiply and divide 2- and 3-digit numbers by a single-digit number, without remainders for division.
- 3NI 15. Evaluate the reasonableness of computations based upon operations and the numbers involved; e.g., considering relative size, place value and estimates.

3-4NB I. Demonstrate fluency in multiplication facts with factors through 10 and corresponding divisions.
- 3NI 13. Demonstrate fluency in multiplication facts through 10 and corresponding division facts.

3-4NB J. Estimate the results of whole number computations using number computations using a variety of strategies, and judge the reasonableness.
- 3NI 15. Evaluate the reasonableness of computations based upon operations and the numbers involved; e.g., considering relative size, place value and estimates.

3-4NB K. Analyze and solve multi-step problems involving addition, subtraction, multiplication and division of whole numbers.

3-4NB L. Use a variety of methods and appropriate tools (mental math, paper and pencil, calculators) for computing with whole numbers.

3NI 8. Model, represent and explain multiplication; e.g., repeated addition, skip counting, rectangular arrays and area model. For example:

a. Use conventional mathematical symbols to write equations for word problems involving multiplication.

b. Understand that, unlike addition and subtraction, the factors in multiplication and division may have different units; e.g., 3 boxes of 5 cookies each.

3NI 9. Model, represent and explain division; e.g., sharing equally, repeated subtraction, rectangular arrays and area model. For example:

a. Translate contextual situations involving division into conventional mathematical symbols.

b. Explain how a remainder may impact an answer in a real-world situation; e.g., 14 cookies being shared by 4 children.

3-4NB M. Add and subtract commonly used fractions with like denominators and decimals, using models and paper and pencil.

Grade: Three — Measurement Standard

Measurement Standard
Students estimate and measure to a required degree of accuracy and precision by selection and using appropriate units, tools and technologies.

Mathematical Processes Benchmarks Gr. 3 - 4:

3-4MPB A. Apply and justify the use of a variety of problem-solving strategies; e.g., make an organized list, guess and check.
3-4MPB B. Use an organized approach and appropriate strategies to solve multi-step problems.
3-4MPB C. Interpret results in the context of the problem being solved; e.g., the solution must be a whole number of buses when determining the number of buses necessary to transport students.
3-4MPB D. Use mathematical strategies to solve problems that relate to other curriculum areas and the real world; e.g., use a timeline to sequence events; use symmetry in artwork.
3-4MPB E. Link concepts to procedures and to symbolic notation; e.g., model 3 x 4 with a geometric array, represent one-third by dividing an object into three equal parts.
3-4MPB F. Recognize relationships among different topics within mathematics; e.g., the length of an object can be represented by a number.
3-4MPB G. Use reasoning skills to determine and explain the reasonableness of a solution with respect to the problem situation.
3-4MPB H. Recognize basic valid and invalid arguments, and use examples and counter examples, models, number relationships, and logic to support or refute.
3-4MPB I. Represent problem situations in a variety of forms (physical model, diagram, in words or symbols), and recognize when some ways of representing a problem may be more helpful than others.
3-4MPB J. Read, interpret, discuss and write about mathematical ideas and concepts using both everyday and mathematical language.
3-4MPB K. Use mathematical language to explain and justify mathematical ideas, strategies and solutions.

Benchmarks Gr. 3 - 4:
Indicators Gr. 3:

3-4MB A. Select appropriate units for perimeter, area, weight, volume (capacity), time and temperature, using:
- objects of uniform size;
- U.S. customary units; e.g., mile, square inch, cubic inch, second, degree Fahrenheit, and other units as appropriate;
- metric units; e.g., millimeter, kilometer, square centimeter, kilogram, cubic centimeter, degree Celsius, and other units as appropriate.

3MI 1. Identify and select appropriate units for measuring:
a. length – miles, kilometers and other units of length, - meter, yard, foot**,** -measure as appropriate;
b. volume (capacity) – gallons;
c. weight – ounces, pounds, grams, or kilograms;
d. temperature – degrees (Fahrenheit or Celsius).

3-4MB B. Know that the number of units is inversely related to the size of the unit for any item being measured.

3-4MB C. Develop common referents for units of measure for length, weight, volume (capacity) and time to make comparisons and estimates.
3MI 2. Establish personal or common referents to include additional units; e.g., a gallon container of milk; a postage stamp is about a square inch.

3-4MB D. Identify appropriate tools and apply counting techniques for measuring side lengths, perimeter and area of squares, rectangles, and simple irregular two -dimensional shapes, volume of rectangular prisms, and time and temperature.

3MI 4. Read thermometers in both Fahrenheit and Celsius scales.

3MI 5 Estimate and measure length, weight and volume (capacity), using metric and U.S. customary units, accurate to the nearest ½ or ¼ units as appropriate.

3MI 6. Use appropriate measurement tools and techniques to construct a figure or approximate an amount of specified length, weight or volume (capacity); e.g., construct a rectangle with length 2 ½ inches and width 3 inches, fill a measuring cup to the 3/4 cup mark.

3MI 7. Make estimates for perimeter, area and volume using links, tiles, cubes and other models.

3-4MB E. Tell time to the nearest minute.

3MI 3. Tell time to the nearest minute and find elapsed time using a calendar or a clock.

Grade: Three — Geometry and Spatial Sense Standard

Geometry and Spatial Sense Standard
Students identify, classify, compare and analyze characteristics, properties and relationships of one-, two- and three-dimensional geometric figures and objects. Students use spatial reasoning, properties of geometric objects, and transformations to analyze mathematical situations and solve problems.

Mathematical Processes Benchmarks Gr. 3 - 4:

3-4MPB A. Apply and justify the use of a variety of problem-solving strategies; e.g., make an organized list, guess and check.
3-4MPB B. Use an organized approach and appropriate strategies to solve multi-step problems.
3-4MPB C. Interpret results in the context of the problem being solved; e.g., the solution must be a whole number of buses when determining the number of buses necessary to transport students.
3-4MPB D. Use mathematical strategies to solve problems that relate to other curriculum areas and the real world; e.g., use a timeline to sequence events; use symmetry in artwork.
3-4MPB E. Link concepts to procedures and to symbolic notation; e.g., model 3 x 4 with a geometric array, represent one-third by dividing an object into three equal parts.
3-4MPB F. Recognize relationships among different topics within mathematics; e.g., the length of an object can be represented by a number.
3-4MPB G. Use reasoning skills to determine and explain the reasonableness of a solution with respect to the problem situation.
3-4MPB H. Recognize basic valid and invalid arguments, and use examples and counter examples, models, number relationships, and logic to support or refute.
3-4MPB I. Represent problem situations in a variety of forms (physical model, diagram, in words or symbols), and recognize when some ways of representing a problem may be more helpful than others.
3-4MPB J. Read, interpret, discuss and write about mathematical ideas and concepts using both everyday and mathematical language.
3-4MPB K. Use mathematical language to explain and justify mathematical ideas, strategies and solutions.

Benchmarks Gr. 3-4:
Indicators Gr. 3:

3-4GSB A. Provide rationale for groupings and comparisons of two-dimensional figures and three-dimensional objects.

3-4GSB B. Describe and identify points, lines and planes in the environment.

3-4GSB C. Describe and identify intersecting, parallel and perpendicular lines or segments in the environment.

3-4GSB D. Identify and draw right, obtuse, acute and straight angles.
3GSI 2. Identify and describe the relative size of angles with respect to right angles as follows:
- Use physical models, like straws, to make different sized angles by opening and closing the sides, not by changing the side lengths.
- Identify, classify and draw right, acute, obtuse and straight angles.

3-4GSB E. Use attributes to describe, classify and sketch plane figures and build solid objects.

3GSI 1. Analyze and describe properties of two-dimensional shapes and three-dimensional objects using terms such as vertex, edge, angle, side and face.

3GSI 5. Build a three-dimensional model of an object composed of cubes; e.g., construct a model based on an illustration or actual object.

3-4GSB F. Develop definitions of classes of shapes.

3-4GSB G. Find and name locations in coordinate systems.

3GSI 3. Find and name locations on a labeled grid or coordinate system; e.g., a map or graph.

3-4GSB H. Identify and describe line and rotational symmetry in two-dimensional shapes and designs.

3GSI 4. Draw lines of symmetry to verify symmetrical two-dimensional shapes.

3-4GSB I. Describe, identify and model reflections, rotations and translations, using physical materials.

3-4GSB J. Describe a motion or series of transformations that show two shapes are congruent.

Grade: Three — Patterns, Functions and Algebra Standard

Patterns, Functions and Algebra Standard
Students use patterns, relations and functions to model, represent and analyze problem situations that involve variable quantities. Students analyze, model and solve problems using various representations such as tables, graphs and equations.

Mathematical Processes Benchmarks Gr. 3 - 4:

- 3-4MPB A. Apply and justify the use of a variety of problem-solving strategies; e.g., make an organized list, guess and check.
- 3-4MPB B. Use an organized approach and appropriate strategies to solve multi-step problems.
- 3-4MPB C. Interpret results in the context of the problem being solved; e.g., the solution must be a whole number of buses when determining the number of buses necessary to transport students.
- 3-4MPB D. Use mathematical strategies to solve problems that relate to other curriculum areas and the real world; e.g., use a timeline to sequence events; use symmetry in artwork.
- 3-4MPB E. Link concepts to procedures and to symbolic notation; e.g., model 3 x 4 with a geometric array, represent one-third by dividing an object into three equal parts.
- 3-4MPB F. Recognize relationships among different topics within mathematics; e.g., the length of an object can be represented by a number.
- 3-4MPB G. Use reasoning skills to determine and explain the reasonableness of a solution with respect to the problem situation.
- 3-4MPB H. Recognize basic valid and invalid arguments, and use examples and counter examples, models, number relationships, and logic to support or refute.
- 3-4MPB I. Represent problem situations in a variety of forms (physical model, diagram, in words or symbols), and recognize when some ways of representing a problem may be more helpful than others.
- 3-4MPB J. Read, interpret, discuss and write about mathematical ideas and concepts using both everyday and mathematical language.
- 3-4MPB K. Use mathematical language to explain and justify mathematical ideas, strategies and solutions.

Benchmarks Gr. 3-4:
Indicators Gr. 3:

- 3-4PFAB A. Analyze and extend patterns, and describe the rule in words.
  - 3PFAI 1. Extend multiplicative and growing patterns, and describe the pattern or rule in words.
  - 3PFAI 2. Analyze and replicate arithmetic sequences with and without a calculator.
- 3-4PFAB B. Use patterns to make predictions, identify relationships, and solve problems.
  - 3PFAI 3. Use patterns to make predictions, identify relationships, and solve problems.
- 3-4PFAB C. Write and solve open sentences and explain strategies.
  - 3PFAI 5. Write, solve and explain simple mathematical statements, such as $7 + \square > 8$ or $\triangle + 8 = 10$.
  - 3PFAI 6. Express mathematical relationships as equations and inequalities.
- 3-4PFAB D. Represent an unknown quantity as a variable using a symbol, including letters.
- 3-4PFAB E. Use variables to create and solve equations representing problem situations.
  - 3PFAI 4. Model problem situations using objects, pictures, tables, numbers, letters and other symbols.

3-4PFAB	F.	Construct and use a table of values to solve problems associated with mathematical relationships.
		3PFAI 7. Create tables to record, organize and analyze data to discover patterns and rules.
3-4PFAB	G.	Describe how a change in one variable affects the value of a related variable.
		3PFAI 8. Identify and describe quantitative changes, especially those involving addition and subtraction; e.g., the height of water in a glass becoming 1 centimeter lower each week due to evaporation.

Grade: Three — Data Analysis and Probability Standard

Data Analysis and Probability Standard:
Students pose questions and collect, organize, represent, interpret and analyze data to answer those questions. Students develop and evaluate inferences, predictions and arguments that are based on data.

Mathematical Processes Benchmarks Gr. 3 - 4:

- 3-4MPB A. Apply and justify the use of a variety of problem-solving strategies; e.g., make an organized list, guess and check.
- 3-4MPB B. Use an organized approach and appropriate strategies to solve multi-step problems.
- 3-4MPB C. Interpret results in the context of the problem being solved; e.g., the solution must be a whole number of buses when determining the number of buses necessary to transport students.
- 3-4MPB D. Use mathematical strategies to solve problems that relate to other curriculum areas and the real world; e.g., use a timeline to sequence events; use symmetry in artwork.
- 3-4MPB E. Link concepts to procedures and to symbolic notation; e.g., model 3 x 4 with a geometric array, represent one-third by dividing an object into three equal parts.
- 3-4MPB F. Recognize relationships among different topics within mathematics; e.g., the length of an object can be represented by a number.
- 3-4MPB G. Use reasoning skills to determine and explain the reasonableness of a solution with respect to the problem situation.
- 3-4MPB H. Recognize basic valid and invalid arguments, and use examples and counter examples, models, number relationships, and logic to support or refute.
- 3-4MPB I. Represent problem situations in a variety of forms (physical model, diagram, in words or symbols), and recognize when some ways of representing a problem may be more helpful than others.
- 3-4MPB J. Read, interpret, discuss and write about mathematical ideas and concepts using both everyday and mathematical language.
- 3-4MPB K. Use mathematical language to explain and justify mathematical ideas, strategies and solutions.

Benchmarks Gr. 3-4:
Indicators Gr. 3:

- 3-4DPB A. Gather and organize data from surveys and classroom experiments, including data collected over a period of time.
  - 3DPI 1. Collect and organize data from an experiment, such as recording and classifying observations or measurements, in response to a question posed.

- 3-4DPB B. Read and interpret tables, charts, graphs (bar, picture, line, line plot), and timelines as sources of information, identify main idea, draw conclusions, and make predictions.
  - 3DPI. 4. Support a conclusion or prediction orally and in writing, using information in a table or graph.
  - 3DPI 5. Match a set of data with a graphical representation of the data.
  - 3DPI 7. Analyze and interpret information represented on a timeline.

- 3-4DPB C. Construct charts, tables and graphs to represent data, including picture graphs, bar graphs, line graphs, line plots and Venn diagrams.
  - 3DPI 6. Translate information freely among charts, tables, line plots, picture graphs and bar graphs; e.g., create a bar graph from the information in a chart.

- 3-4DPB D. Read, interpret and construct graphs in which icons represent more than a single unit or intervals greater than one; e.g., each 🚲 = 10 bicycles or the

intervals on an axis are multiples of 10.

3DPI 2. Draw and interpret picture graphs in which a symbol or picture represents more than one object.

3DPI 3. Read, interpret and construct bar graphs with intervals greater than one.

3-4DPB E. Describe data using mode, median, range.

3DPI 8. Identify the mode of a data set and describe the information it gives about a data set.

3-4DPB F. Conduct a simple probability experiment and draw conclusions about the likelihood of possible outcomes.

3DPI 9. Conduct a simple experiment or simulation of a simple event, record the results in a chart, table or graph, and use the results to draw conclusions about the likelihood of possible outcomes.

3-4DPB G. Identify and represent possible outcomes, such as arrangements of a set of up to four members and possible combinations from several sets, each containing 2 or 3 members.

3DPI 10. Use physical models, pictures, diagrams and lists to solve problems involving possible arrangements or combinations of two to four objects.

3-4DPB H. Use the set of possible outcomes to describe and predict events.

Grade: Four — Number, Number Sense and Operations Standards

Number, Number Sense Standard:
Students demonstrate number sense including an understanding of number systems and operations, and how they relate to one another. Students compute fluently and make reasonable estimates using paper and pencil, technology-supported and mental methods.

Mathematical Processes Benchmarks 3 - 4:

3-4MPB	A.	Apply and justify the use of a variety of problem-solving strategies; e.g., make an organized list, guess and check.
3-4MPB	B.	Use an organized approach and appropriate strategies to solve multi-step problems.
3-4MPB	C.	Interpret results in the context of the problem being solved; e.g., the solution must be a whole number of buses when determining the number of buses necessary to transport students.
3-4MPB	D.	Use mathematical strategies to solve problems that relate to other curriculum areas and the real world; e.g., use a timeline to sequence events; use symmetry in artwork.
3-4MPB	E.	Link concepts to procedures and to symbolic notation; e.g., model 3 x 4 with a geometric array, represent one-third by dividing an object into three equal parts.
3-4MPB	F.	Recognize relationships among different topics within mathematics; e.g., the length of an object can be represented by a number.
3-4MPB	G.	Use reasoning skills to determine and explain the reasonableness of a solution with respect to the problem situation.
3-4MPB	H.	Recognize basic valid and invalid arguments, and use examples and counter examples, models, number relationships, and logic to support or refute.
3-4MPB	I.	Represent problem situations in a variety of forms (physical model, diagram, in words or symbols), and recognize when some ways of representing a problem may be more helpful than others.
3-4MPB	J.	Read, interpret, discuss and write about mathematical ideas and concepts using both everyday and mathematical language.
3-4MPB	K.	Use mathematical language to explain and justify mathematical ideas, strategies and solutions.

Benchmarks Gr. 3-4:
Indicators Gr. 4:

3-4NB	A.	Use place value structure of the base-ten number system to read, write, represent and compare whole numbers and decimals.		
		4NI	2.	Use place value structure of the base-ten number system to read, write, represent and compare whole numbers through millions and decimals through thousandths.
		4NI	3.	Round whole numbers to a given place value.
3-4NB	B.	Recognize and generate equivalent representations for whole numbers, fractions and decimals.		
		4NI	1.	Identify and generate equivalent forms of fractions and decimals. For example:
			a.	Connect physical, verbal and symbolic representations of fractions, decimals and whole numbers; e.g., ½, 5/10, "five tenths", 0.5, shaded rectangles with half, and five tenths.
			b.	Understand and explain that ten tenths is the same as one whole in both fraction and decimal form.
3-4NB	C.	Represent commonly used fractions and mixed numbers using words and physical models.		
3-4NB	D.	Use models, points of reference and equivalent forms of commonly used fractions to judge the size of fractions and to compare, describe and		

order them.

4NI 5. Use models and points of reference to compare commonly used fractions.

3-4NB E. Recognize and classify numbers as prime or composite and list factors.

4NI 4. Identify and represent factors and multiples of whole numbers through 100, and classify numbers as prime or composite.

3-4NB F. Count money and make change using both coins and paper bills.

4NI 8. Solve problems involving counting money and making change, using both coins and paper bills.

3-4NB G. Model and use commutative and associative properties for addition and multiplication.

3-4NB H. Use relationships between operations, such as subtraction as the inverse of addition and division as the inverse of multiplication.

3-4NB I. Demonstrate fluency in multiplication facts with factors through 10 and corresponding division facts.

3-4NB J. Estimate the results of whole number computations using number computations using a variety of strategies, and judge the reasonableness.

3-4NB K. Analyze and solve multi-step problems involving addition, subtraction, multiplication and division of whole numbers.

4NI 6. Use associative and distributive properties to simplify and perform computations; e.g., use left to right multiplication and the distributive property to find exact answers without paper and pencil, such as 5 x 47 = 5 x 40 + 5 x 7 = 200 + 35 = 235.

4NI 7. Recognize that division may be used to solve different types of problem situations and interpret the meaning of remainders; e.g., situations involving measurement, and money.

4NI 12. Analyze and solve multi-step problems involving addition, subtraction, multiplication and division using an organized approach, and verify and interpret results with respect to the original problem.

3-4NB L. Use a variety of methods and appropriate tools (mental math, paper and pencil, calculators) for computing with whole numbers.

4NI 11. Develop and explain strategies for performing computations mentally.

4NI 13. Use a variety of methods and appropriate tools for computing with whole numbers; e.g., mental math, paper and pencil, and calculator.

4NI 14. Demonstrate fluency in adding and subtracting whole numbers and in multiplying and dividing whole numbers by 1- and 2-digit numbers and multiples of ten.

3-4NB M. Add and subtract commonly used fractions with like denominators and decimals, using models and paper and pencil.

4NI 9. Estimate the results of computations involving whole numbers, fractions and decimals, using a variety of strategies.

4NI 10. Use physical models, visual representations, and paper and pencil to add and subtract decimals and commonly used fractions with like denominators.

Grade: Four — Measurement Standard

Measurement Standard
Students estimate and measure to a required degree of accuracy and precision by selection and using appropriate units, tools and technologies.

Mathematical Processes Benchmarks 3 - 4:

3-4MPB	A.	Apply and justify the use of a variety of problem-solving strategies; e.g., make an organized list, guess and check.
3-4MPB	B.	Use an organized approach and appropriate strategies to solve multi-step problems.
3-4MPB	C.	Interpret results in the context of the problem being solved; e.g., the solution must be a whole number of buses when determining the number of buses necessary to transport students.
3-4MPB	D.	Use mathematical strategies to solve problems that relate to other curriculum areas and the real world; e.g., use a timeline to sequence events; use symmetry in artwork.
3-4MPB	E.	Link concepts to procedures and to symbolic notation; e.g., model 3 x 4 with a geometric array, represent one-third by dividing an object into three equal parts.
3-4MPB	F.	Recognize relationships among different topics within mathematics; e.g., the length of an object can be represented by a number.
3-4MPB	G.	Use reasoning skills to determine and explain the reasonableness of a solution with respect to the problem situation.
3-4MPB	H.	Recognize basic valid and invalid arguments, and use examples and counter examples, models, number relationships, and logic to support or refute.
3-4MPB	I.	Represent problem situations in a variety of forms (physical model, diagram, in words or symbols), and recognize when some ways of representing a problem may be more helpful than others.
3-4MPB	J.	Read, interpret, discuss and write about mathematical ideas and concepts using both everyday and mathematical language.
3-4MPB	K.	Use mathematical language to explain and justify mathematical ideas, strategies and solutions.

Benchmarks Gr. 3-4:
Indicators Gr. 4:

3-4MB A. Select appropriate units for perimeter, area, weight, volume (capacity), time and temperature, using:
- objects of uniform size;
- U.S. customary units; e.g., mile, square inch, cubic inch, second, degree Fahrenheit, and other units as appropriate;
- metric units; e.g., millimeter, kilometer, square centimeter, kilogram, cubic centimeter, degree Celsius, and other units as appropriate.

3-4MB B. Know that the number of units is inversely related to the size of the unit for any item being measured.

4MI	1.	Relate the number of units to the size of the units used to measure an object; e.g., compare the number of cups to fill a pitcher to the number of quarts to fill the same pitcher.
4MI	2.	Demonstrate and describe perimeter as surrounding and area as covering a two-dimensional shape, and volume as filling a three-dimensional object.

3-4MB C. Develop common referents for units of measure for length, weight, volume (capacity) and time to make comparisons and estimates.

4MI 3. Identify and select appropriate units to measure:
a. perimeter – string or links (inches or centimeters).
b. area – tiles (square inches or square centimeters).
c. volume – cubes (cubic inches or cubic centimeters).

3-4MB D. Identify appropriate tools and apply counting techniques for measuring side lengths, perimeter and area of squares, rectangles, and simple irregular two -dimensional shapes, volume of rectangular prisms, and time and temperature.

4MI 4. Develop and use strategies to find perimeter using string or links, area using tiles or a grid, and volume using cubes; e.g., count squares to find area of regular or irregular shapes on a grid, layer cubes in a box to find its volume.

4MI 5. Make simple unit conversions within a measurement system; e.g., inches to feet, kilograms to grams, quarts to gallons.

4MI 6. Write, solve and verify solutions to multi-step problems involving measurement.

3-4MB E. Tell time to the nearest minute.

Grade: Four — Geometry and Spatial Sense Standard

Geometry and Spatial Sense Standard
Students identify, classify, compare and analyze characteristics, properties and relationships of one-, two- and three-dimensional geometric figures and objects. Students use spatial reasoning, properties of geometric objects, and transformations to analyze mathematical situations and solve problems.

Mathematical Processes Benchmarks 3 - 4:

3-4MPB	A.	Apply and justify the use of a variety of problem-solving strategies; e.g., make an organized list, guess and check.
3-4MPB	B.	Use an organized approach and appropriate strategies to solve multi-step problems.
3-4MPB	C.	Interpret results in the context of the problem being solved; e.g., the solution must be a whole number of buses when determining the number of buses necessary to transport students.
3-4MPB	D.	Use mathematical strategies to solve problems that relate to other curriculum areas and the real world; e.g., use a timeline to sequence events; use symmetry in artwork.
3-4MPB	E.	Link concepts to procedures and to symbolic notation; e.g., model 3 x 4 with a geometric array, represent one-third by dividing an object into three equal parts.
3-4MPB	F.	Recognize relationships among different topics within mathematics; e.g., the length of an object can be represented by a number.
3-4MPB	G.	Use reasoning skills to determine and explain the reasonableness of a solution with respect to the problem situation.
3-4MPB	H.	Recognize basic valid and invalid arguments, and use examples and counter examples, models, number relationships, and logic to support or refute.
3-4MPB	I.	Represent problem situations in a variety of forms (physical model, diagram, in words or symbols), and recognize when some ways of representing a problem may be more helpful than others.
3-4MPB	J.	Read, interpret, discuss and write about mathematical ideas and concepts using both everyday and mathematical language.
3-4MPB	K.	Use mathematical language to explain and justify mathematical ideas, strategies and solutions.

Benchmarks Gr. 3-4:
Indicators Gr. 4:

3-4GSB	A.	Provide rationale for groupings and comparisons of two-dimensional figures and three-dimensional objects.
3-4GSB	B.	Describe and identify points, lines and planes in the environment. 4GSI 5. Describe points, lines and planes, and identify models in the environment.
3-4GSB	C.	Describe and identify intersecting, parallel and perpendicular lines or segments in the environment. 4GSI 1. Identify, describe and model intersecting, parallel and perpendicular lines and line segments; e.g., use straws or other material to model lines.
3-4GSB	D.	Identify and draw right, obtuse, acute and straight angles
3-4GSB	E.	Use attributes to describe, classify and sketch plane figure and build solid objects. 4GSI 2. Describe, classify, compare and model two- and three-dimensional objects using their attributes.
3-4GSB	F.	Develop definitions of classes of shapes 4GSI 3. Identify similarities and differences of quadrilaterals; e.g., squares, rectangles, parallelograms and trapezoids.

		4GSI	4.	Identify and define triangles based on angle measures (equiangular, right, acute and obtuse triangles) and side lengths (isosceles, equilateral and scalene triangles).
3-4GSB	G.	Find and name locations in coordinate systems		
		4GSI	6.	Specify locations and plot ordered pairs on a coordinate plane, using first quadrant points.
3-4GSB	H.	Identify and describe line and rotational symmetry in two-dimensional shapes and designs.		
3-4GSB	I.	Describe, identify and model reflections, rotations and translations, using physical materials.		
3-4GSB	J.	Describe a motion or series of transformations that show two shapes are congruent.		
		4GSI	7.	Identify, describe and use reflections (flips), rotations (turns), and translations (slides) in solving geometric problems; e.g., use transformations to determine if 2 shapes are congruent.
		4GSI	8.	Use geometric models to solve problems in other areas of mathematics, such as number (multiplication/division) and measurement (area, perimeter, border).

Grade: Four — Patterns, Functions and Algebra Standard

Patterns, Functions and Algebra Standard
Students use patterns, relations and functions to model, represent and analyze problem situations that involve variable quantities**.** Students analyze, model and solve problems using various representations such as tables, graphs and equations.

Mathematical Processes Benchmarks 3 - 4:

3-4MPB	A.	Apply and justify the use of a variety of problem-solving strategies; e.g., make an organized list, guess and check.
3-4MPB	B.	Use an organized approach and appropriate strategies to solve multi-step problems.
3-4MPB	C.	Interpret results in the context of the problem being solved; e.g., the solution must be a whole number of buses when determining the number of buses necessary to transport students.
3-4MPB	D.	Use mathematical strategies to solve problems that relate to other curriculum areas and the real world; e.g., use a timeline to sequence events; use symmetry in artwork.
3-4MPB	E.	Link concepts to procedures and to symbolic notation; e.g., model 3 x 4 with a geometric array, represent one-third by dividing an object into three equal parts.
3-4MPB	F.	Recognize relationships among different topics within mathematics; e.g., the length of an object can be represented by a number.
3-4MPB	G.	Use reasoning skills to determine and explain the reasonableness of a solution with respect to the problem situation.
3-4MPB	H.	Recognize basic valid and invalid arguments, and use examples and counter examples, models, number relationships, and logic to support or refute.
3-4MPB	I.	Represent problem situations in a variety of forms (physical model, diagram, in words or symbols), and recognize when some ways of representing a problem may be more helpful than others.
3-4MPB	J.	Read, interpret, discuss and write about mathematical ideas and concepts using both everyday and mathematical language.
3-4MPB	K.	Use mathematical language to explain and justify mathematical ideas, strategies and solutions.

Benchmarks Gr. 3-4:
Indicators Gr. 4:

3-4PFAB	A.	Analyze and extend patterns, and describe the rule in words.
3-4PFAB	B.	Use patterns to make predictions, identify relationships, and solve problems. 4PFAI 1. Use models and words to describe, extend and make generalizations of patterns and relationships occurring in computation, numerical patterns, geometry, graphs and other applications.
3-4PFAB	C.	Write and solve open sentences and explain strategies 4PFAI 5. Represent mathematical relationships with equations or inequalities.
3-4PFAB	D.	Represent an unknown quantity as a variable using a symbol, including letters. 4PFAI 2. Represent and analyze patterns and functions using words, tables and graphs.
3-4PFAB	E.	Use variables to create and solve equations representing problem situations. 4PFAI 4. Use rules and variables to describe patterns and other relationships.

3-4PFAB	F.	Construct and use a table of values to solve problems associated with mathematical relationships. 4PFAI 3. Construct a table of values to solve problems associated with a mathematical relationship.
3-4PFAB	G.	Describe how a change in one variable affects the value of a related variable. 4PFAI 6. Describe how a change in one variable affects the value of a related variable; e.g., as one increases the other increases or as one increases the other decreases.

Grade: Four — Data Analysis and Probability Standard

Data Analysis and Probability Standard:
Students pose questions and collect, organize, represent, interpret and analyze data to answer those questions. Students develop and evaluate inferences, predictions and arguments that are based on data.

Mathematical Processes Benchmarks 3 - 4:

3-4MPB	A.	Apply and justify the use of a variety of problem-solving strategies; e.g., make an organized list, guess and check.
3-4MPB	B.	Use an organized approach and appropriate strategies to solve multi-step problems.
3-4MPB	C.	Interpret results in the context of the problem being solved; e.g., the solution must be a whole number of buses when determining the number of buses necessary to transport students.
3-4MPB	D.	Use mathematical strategies to solve problems that relate to other curriculum areas and the real world; e.g., use a timeline to sequence events; use symmetry in artwork.
3-4MPB	E.	Link concepts to procedures and to symbolic notation; e.g., model 3 x 4 with a geometric array, represent one-third by dividing an object into three equal parts.
3-4MPB	F.	Recognize relationships among different topics within mathematics; e.g., the length of an object can be represented by a number.
3-4MPB	G.	Use reasoning skills to determine and explain the reasonableness of a solution with respect to the problem situation.
3-4MPB	H.	Recognize basic valid and invalid arguments, and use examples and counter examples, models, number relationships, and logic to support or refute.
3-4MPB	I.	Represent problem situations in a variety of forms (physical model, diagram, in words or symbols), and recognize when some ways of representing a problem may be more helpful than others.
3-4MPB	J.	Read, interpret, discuss and write about mathematical ideas and concepts using both everyday and mathematical language.
3-4MPB	K.	Use mathematical language to explain and justify mathematical ideas, strategies and solutions.

Benchmarks Gr. 3-4:
Indicators Gr. 4:

3-4DPBA. Gather and organize data from surveys and classroom experiments, including data collected over a period of time.
- 4DPI 1. Create a plan for collecting data for a specific purpose.

3-4DPBB. Read and interpret tables, charts, graphs (bar, picture, line, line plot), and timelines as sources of information, identify main idea, draw conclusions, and make predictions.
- 4DPI 5. Propose and explain interpretations and predictions based on data displayed in tables, charts and graphs.

3-4DPB C. Construct charts, tables and graphs to represent data, including picture graphs, bar graphs, line graphs, line plots and Venn diagrams.
- 4DPI 2. Represent and interpret data using tables, bar graphs, line plots and line graphs.
- 4DPI 3. Interpret and construct Venn diagrams to sort and describe data.
- 4DPI 4. Compare different representations of the same data to evaluate how well each representation shows important aspects of the data, and identify appropriate ways to display the data.

3-4DPB D. Read, interpret ad construct graphs in which icons represent more than a single unit or interval greater than one; e.g. each  = 10 bicycles or the

intervals on an axis are multiples of 10.

3-4DPB E. Describe data using mode, median and range.

- 4DPI 6. Describe the characteristics of a set of data based on a graphical representation, such as range of the data, clumps of data, and holes in the data.
- 4DPI 7. Identify the median of a set of data and describe what it indicates about the data.
- 4DPI 8. Use range, median and mode to make comparisons among related sets of data.

3-4DPB F. Conduct a simple probability experiment and draw conclusions about the likelihood of possible outcomes.

- 4DPI 9. Conduct simple probability experiments and draw conclusions from the results; e.g., rolling number cubes or drawing marbles from a bag.
- 4SPI 10. Represent the likelihood of possible outcomes for chance situations; e.g., probability of selecting a red marble from a bag containing 3 red and 5 white marbles.
- 4DPI 11. Relate the concepts of impossible and certain-to-happen events to the numerical values of 0 (impossible) and 1 (certain).
- 4DPI 12. Place events in order of likelihood and use a diagram or appropriate language to compare the chance of each event occurring; e.g., impossible, unlikely, equal, likely, certain.

3-4DPB G. Identify and represent possible outcomes, such as arrangements of a set of up to four members and possible combinations from several sets, each containing 2 or 3 members.

3-4DPB H. Use the set of possible outcomes to describe and predict events.

- 4DPI 10. Represent the likelihood of possible outcomes for chance situations; e.g., probability of selecting a red marble from a bag containing 3 red and 5 white marbles.
- 4DPI 11. Relate the concepts of impossible and certain-to-happen events to the numerical values of 0 (impossible) and 1 (certain).
- 4DPI 13. List and count all possible combinations using one member from each of several sets, each containing 2 or 3 members; e.g., the number of possible outfits from 3 shirts, 2 shorts and 2 pairs of shoes.

RESEARCH BASE: NUMBERS AND OPERATIONS
GRADES 5 -7

INTRODUCTION:

Students in grades 5 – 7 must develop number sense, computational estimation, mental computation, and number size in order to thoroughly understand the real number system. The primary focus in the middle grades should be on fractions, decimals, percents, integers, and rational numbers. Students should apply their understanding of factors, multiples, and prime factorization to problems involving fractions. Students need to develop an understanding of decimals as fractions whose denominators are powers of 10. The concept of fractions should be extended to include rates, ratios, and proportionality. Percents can be thought about in ways that combine aspects of both fractions and decimals, paying particular attention to percents less than 1 or greater than 100. Applications with integers will develop the notation that they represent relative changes in values. As a result of the studies in numbers and operations, students will be able to judge the advantages and disadvantages of various representations of numbers.
Students in middle grades must also understand the meaning of operations and how they relate to one another. In addition to developing proficiency with fraction, decimal, percent, integer, and rational number computations, students should be able to determine the reasonableness of their answers. Technologies such as calculations and computers can aid in connecting basic skills and calculation procedures to a deeper mathematical understanding. Students should also have experiences solving problems in context, choosing the appropriate computational methods, and deciding whether to use approximate or exact values.

RESEARCH BASE:

Middle School students are able to identify the place values of the digits that appear in a number, but they cannot use the knowledge confidently in context (Sowder 1992a).

Upper elementary- and middle-school students often to dot understand that decimal fractions represent concrete objects that can be measured by units, tenths of units, hundredths of units, and so on (Hiebert, 1992). Other students have little understanding of the value represented by each of the digits of a decimal number or know the value of the number is the sum of the value of its digits. Students of all ages have problems choosing the largest or the smallest in a set of decimals with different numbers of digits to the right of the decimal points (Carpenter et al., 1981, Hiebert & Wearne, 1986; Resnick et al, 1989). Upper elementary- and middle-school students may exhibit limited understanding of the meaning of fractional number (Kieren, 1992). From their experience with whole numbers, many students appear to develop a belief that "multiplication makes bigger and division makes smaller". When students solve problems in which they need to decide whether to multiply or divide fractions or decimals, this belief has negative consequences that have been well researched (Greer, 1992). Also a mistaken expectation about the magnitude of a computational result is likely to interfere with students' making sense of multiplication and division of fractions or decimals (Gaeber & Tannenhaus, 1993). For example, fewer than one-third of the thirteen-year-old U. S. students tested in the National Assessment of Education Progress (NAEP) in 1988 correctly chose the largest number from 3/4, 9/16, 5/8, and 2/3 (Kouba, Carpenter, and Swafford, 1989), Students' difficulties with comparison of fractions have also been documented in more recent NAEP administrations (Kouba, Zawojewski, and Strutchens, 1997).

Students are allowed much more flexibility in mathematics with the use of integers, which can be thought of in terms of a number line (AAAS, p.131). They can now analyze numbers in terms of below sea level, debt, and left of zero on the real number line.
Middle-school and even high-school students may be limited understanding about the nature and purpose of estimation. They often think it is inferior to exact computation and equate it with guessing (Sowder, 1992b), so they do not believe estimation is useful (Sowder & Wheeler, 1989). Students who see estimation as a valuable tactic for obtaining information use estimation more frequently and successfully (Threadgill-Sowder, 1984).

5 – 7 Number and Number Sense

SMART ESSENTIAL FOCUS:

1. Students should know how to compare and order fractions, decimals and percents ($5/6^{th}$).
2. Students should know how to convert among fractions, decimals and percents ($5/6^{th}$).
3. Students should know how to compute fluently with fractions and decimals ($5/6^{th}$).
4. Students should know how to compute with integers and percents (7^{th}).

RESEARCH BASE: MEASUREMENT
GRADES 5 - 7

INTRODUCTION: It should be recognized that students bring to the middle grades many diverse experiences from prior classroom instruction and life experiences. Important aspects of measurement at this level should include choosing and using appropriate units for attributes being measured, estimating measurements, solving problems involving perimeter, area, surface area and volume. In addition, students should become proficient in using measurement tools while working within both metric and customary measurement systems.

Students should become proficient in composing and decomposing two- and three- dimensional shapes in order to find lengths, areas, and volumes of complex objects. Through these investigations, students can discover formulas and use them to solve problems involving perimeter, area, and volume. Students should explore the effect on perimeter and area when dimensions are proportionately changed.

Measurement concepts should be used throughout the school year by providing connections to other mathematics strands. Many measurement topics are closely related to what students learn in geometry.

RESEARCH BASE: Most students' estimation skills are not well developed, especially for metric units; only 30% of 13-yr. olds could estimate the length of a segment to the nearest centimeter (Carpenter, Corbitt, Kepner, Lindquist, & Reys, 1981). About 70% of seventh graders could choose the best estimate of the height of a tall man in feet, but only about 35% could do so in meters. More experience with estimation in both systems of measurement appears to be needed. Development of measurement formulas is an important part of middle grade mathematics. Formulas should be a product of explorations and discovery. This is appropriate at middle grades for concepts like area and perimeter, if students have spent time measuring these in their own ways. Among seventh graders, 56% could compute the area of a rectangle with length and width given, but only 46% could compute the area of the same rectangle drawn on grid paper without the dimensions written in. Only 33 % of students could computer the volume of a rectangular solid with dimensions written in. (Lindquist & Kouba, 1989).

5-7 Measurement
SMART ESSENTIAL FOCUS:

1. Students should know how to select appropriate units to measure perimeter, area, and volume (6^{th}).
2. Students should know how to find area and perimeter using formulas. (6^{th}).
3. Students should know how to use proportional reasoning to make conversions within each measurement system. (7^{th}).

RESEARCH BASE: GEOMETRY
GRADES 5 - 7

INTRODUCTION: Students come to the middle grades with an informal knowledge about geometric concepts. They have had experience in visualizing and drawing lines, angles, triangles and other polygons. Moreover, they have developed intuitive notions about geometry from interacting with objects in their daily lives. Middle school geometry programs should allow students to investigate relationships by drawing, measuring, visualizing, comparing, transforming and classifying two-dimensional and three-dimensional geometric objects. Geometry provides opportunities for development of mathematical reasoning (inductive and deductive), making and validating conjectures, and investigating properties that lead to the classification of geometric shapes. In the middle grades, students begin to use the coordinate plane to investigate transformations, congruence, similarity and symmetry.

Many topics included in the measurement strand are closely connected to the study of geometry.

RESEARCH BASE: Students advance through levels of thought in geometry. Van Hiele has characterized them as visual, descriptive, abstract/relational, and formal deduction (Van Hiele, 2986; Clements & Battista, 1992). At the first level, students identify shapes and figures according to their concrete examples. For example, a student may say that a figure is a rectangle because it looks like a door. At the second level, students identify shapes according to their properties, and here a student might think of a rhombus as a figure with four equal sides. At the third level, students can identify relationships between classes of figures (e.g., a square is a rectangle) and can discover properties of classes of figures by simple logical deduction.
Progress from one of Van Hiele's levels to the next is more dependent upon instruction than age. Given traditional instruction, middle-school students perform at levels one or two (Clements & Battista, 1992). Some evidence suggests it is possible for students to understand the abstract properties of geometric figures by the 5th grade (Clements & Battista, 1989, 1990, 1992; Wirszup, 1976).

Since learning geometry requires students to recognize figures, their properties, and their relationships, spatial visualization skills are essential and contribute in an important way to the learning process. Professional educators point out the need to equip students with mathematical methods that support the full range of problem solving. These methods include the use of imagery, visualization, and spatial concepts and emphasize activities that use concrete representations to improve the perception of spatial relationships. (Lappan & Schram, 1989; Young, 1982). Many researches have hypothesized the differences in students' spatial visualization skills are one cause of their problem solving difficulties. Major findings indicated a strong correlation between spatial ability and problem-solving performance, suggesting that spatial visualization skill is a good predictor of general problem solving (Tillotson, 1985).

Transformations bring a spatial-visual aspect to geometry that is as important as logical-deductive aspects. Also, transformation geometry has important real-world applications such as fabric patterns, mirrors, symmetry in nature, photos and enlargements (Sanok, 1987).

5-7 Geometry
SMART ESSENTIAL FOCUS:

1. Students should know how to describe sizes, positions, and orientations of shapes after rotations, dilations, reflections and translations. (6th).
2. Students should know how to describe and classify fundamental relationships among groups or three-dimensional shapes (7th).
3. Students should know how to identify and plot points on a rectangular coordinate plane in all four quadrants (7th).

RESEARCH BASE: PATTERNS, FUNCTIONS & ALGEBRA STRAND
GRADES 5 – 7

INTRODUCTION: Algebra and algebraic thinking are fundamental to the basic education of all middle school students. Algebraic thinking is a natural extension of arithmetical thinking, but while arithmetic is effective in describing static pictures of the world, algebra is dynamic and a necessary vehicle for describing a changing world. Students in middle grades should investigate patterns expressed in tables, graphs, words, or symbols, with the emphasis on patterns that exhibit linear relationships (constant rate of change). They should explore notions of dependence and independence as the values of variables change in relation to one another. Students should connect this rate of change to the slope of a line and able to interpret its meaning. In addition, they should develop facility in recognizing the equivalence of mathematical representations they can use to transform expressions, to solve problems, and the relate graphical, tabular, and symbolic representations. Students should be given the opportunity to solve linear equations as well as inequalities. Whenever possible, the teaching and learning of algebra in the middle grades should be integrated with other topics in the curriculum.

RESEARCH BASE: Students have difficulty translating between graphical and algebraic representations, especially moving from a graph into an equitation (Leinhardt et al., 1990). Results from the second study of the National Assessment for Educational Progress showed, for instance, that given a line with indicated intercepts, only 5% of 17-year-olds could generate the equation (Carpenter, et al., 1981). Students sometimes resist dealing with multiple representations because they do not find them helpful in solving problems (Dufour-Janvier et al., 1987). Rather they see generating any representation as an end in itself, demanded by the requirements of the teacher or the text rather than by the needs of the problem. In addition, students confound the slope of a graph with the maximum or the minimum value and do not know that the slope of a graph is a measure of the rate (McDermott et all., 1987; Clement, 1989).

5-7 Patterns, Functions and Algebra
ESSENTIAL FOCUS:

1. Students should know how to validate a given solution for an equation or inequality (5^{th})
2. Students should know how to apply formulas in problem solving situations. (6^{th}).
3. Students should know how to relate and compare different forms of representations for a pattern (7^{th}).
4. Students should know that the value of a variable can change (7^{th}).

RESEARCH BASE: DATA ANALYSIS AND PROBABILITY
GRADES 5 - 7

INTRODUCTION: Students in grades 5 - 7 should build on past experiences with data analysis to answer questions about populations and samples. To do this, students should begin to develop and conduct more complex studies. Data collection can be extended to other resources such as websites, and spreadsheets can facilitate data collection and organization. Acquiring new techniques to express the distribution of data will aid in the analysis and interpretation. Interpretation of results should contain appropriate uses of measures of central tendency and spread and construction of lines of best fit. Furthermore, the use of box plots, scatter plots, as well as histograms, circle graphs, and stem-and leaf plots, will facilitate the representation of relationships between two populations.

In the middle grades, students should have numerous opportunities to engage in activities that promote probabilistic thinking. The resulting observations and inferences should be discussed using appropriate terminology. Comparison of theoretical and experimental probability should be undertaken. In addition, students can use and further develop their emerging understanding of proportionality to make predictions about future experiments.

RESEARCH BASE: The concept of the mean is quite difficult for students of all ages to understand even after several years of formal instruction. It is important for students to be able to express ideas about the results of their research in order to see whether it says something useful about the original data (AAAS, p. 19). Several difficulties have been documented in the literature: Students of all ages can talk about the algorithm for computing the mean and relate it to limited contexts, but cannot use it meaningfully in problems (Mokros & Russell, 1992; Pollatsek, Lime & Well, 1981); upper elementary and middle school students believe that the mean of a particular data set is not one precise numerical value but an approximation that can have one of several values (Mokros & Russell, 1992); some middle school students cannot use the mean to a compare two different-sized sets of data (Gal et al., 1990). Research suggests students should be introduced first to location measures that connect with their emerging concept of the "middle", such as the median, and later in the middle school grades, to the mean. Premature introduction of the algorithm for computing the mean divorced from a meaningful context may block students from understanding what averages are for (Mokros & Russell, 1992; Pollatsek et al., 1981).

Students of all ages often interpret graphs of situations as literal pictures rather than as symbolic representations of the situations (Leinhardt, Zaslavsky, Stein, 1990). "In addition, students who found the slope of graph with the maximum or the minimum value do not know that the slope of a graph is a measure of rate" (McDermott, et al., 1987; Clement, 1989). When constructing graphs, middle school students have difficulties with the notions of interval scale and coordinates even after traditional instruction (Kerslake, 1981). "Finally, students read graphs point-by-point and ignore their global feature. This has been attributed to the fact that they are rarely asked questions about maximum and minimum values; intervals over which a function increases, decreases, or levels off; or rates of change" (Herscovics, 1989).

Extensive research points to several misconceptions about probabilistic reasoning that are similar at all age levels and are found among experienced researches (Kahneman, Slovic, & Tversky, 1982; Shaughnessy, 1992). One common misconception is the idea of representativeness, according to which an event is believed to be probable to the extent that it is "typical". For example, many people believe that after a run of heads in coin tossing, tails should be more likely to come up.

5-7 Data Analysis and Probability
ESSENTIAL FOCUS:

1. Students should know how to formulate appropriate questions to investigate relevant issues (6^{th}).
2. Students should know how to find, interpret, and appropriately use measures of central tendency and spread (6th).
3. Students should know the sum of the probabilities of all possible outcomes is one (6^{th}).
4. Students should know how to distinguish between theoretical and experimental probability (7^{th}).

Grade: Five **Number, Number Sense and Operations Standard**

Number, Number Sense Standard: Students demonstrate number sense including an understanding of number systems and operations, and how they relate to one another. Students compute fluently and make reasonable estimates using paper and pencil, technology-supported and mental methods.

Mathematical Processes Benchmarks Gr. 5-7:

5-7MPB	A.	Clarify problem-solving situation and identify potential solution processes; e.g., consider different strategies and approaches to a problem, restate problem from various perspectives.
5-7MPB	B.	Apply and adapt problem-solving strategies to solve a variety of problems, including unfamiliar and non-routine problem situations.
5-7MPB	C.	Use more than one strategy to solve a problem, and recognize there are advantages associated with various methods.
5-7MPB	D.	Recognize whether an estimate or an exact solution is appropriate for a given problem situation.
5-7MPB	E.	Use deductive thinking to construct informal arguments to support reasoning and to justify solutions to problems.
5-7MPB	F.	Use inductive thinking to generalize a pattern of observations for particular cases, make conjectures, and provide supporting arguments for conjectures.
5-7MPB	G.	Relate mathematical ideas to one another and to other content areas; e.g., use area models for adding fractions, interpret graphs in reading, science and social studies.
5-7MPB	H.	Use representations to organize and communicate mathematical thinking and problem solutions.
5-7MPB	I.	Select, apply, and translate among mathematical representations to solve problems; e.g., representing a number as a fraction, decimal or percent as appropriate for a problem
5-7MPB	J.	Communicate mathematical thinking to others and analyze the mathematical thinking and strategies of others.
5-7MPB	K.	Recognize and use mathematical language and symbols when reading, writing and conversing with others.

Benchmarks Gr. 5-7:
Indicators Gr. 5:

5-7NB A. Represent and compare numbers less than 0 through familiar applications and extending the number line.

- 5NI 6. Represent and compare numbers less than 0 by extending the number line and using familiar applications; e.g., temperature, owing money.

5-7NB B. Compare, order and convert among fractions, decimals and percents.

- 5NI 1. Use models and visual representation to develop the concept of ratio as part-to-part and part-to-whole, and the concept of percent as part-to-whole.
- 5NI 2. Use various forms of "one" to demonstrate the equivalence of fractions; e.g., $18/24 = 9/12 \times 2/2 = \frac{3}{4} \times 6/6$.
- 5NI 3. Identify and generate equivalent forms of fractions, decimals and percents.

5-7NB C. Develop meaning for percents, including percents greater than 100 and less than 1.

5-7NB D. Use models and pictures to relate concepts of ratio, proportion and percent.

- 5NI 1. Use models and visual representation to develop the concept of ratio as part-to-part and part-to-whole, and the concept of percent as part-to-whole.

5-7NB E. Use order of operations, including use of parenthesis and exponents to solve multi-step problems, and verify and interpret the results.

5NI 8. Identify and use relationships between operations to solve problems.

5NI 9. Use order of operations, including use of parentheses, to simplify numerical expressions.

5-7NB F. Apply number system properties when performing computations.

5NI 7. Use commutative, associative, distributive, identity and inverse properties to simplify and perform computations.

5-7NB G. Apply and explain the use of common factors, and common multiples in problem situations.

5NI 5. Recognize and identify perfect squares and their roots.

5-7NB H. Use and analyze the steps in standard and non-standard algorithms for computing with fractions, decimals and integers.

5NI 10. Justify why fractions need common denominators to be added or subtracted.

5NI 11. Explain how place value is related to addition and subtraction of decimals; e.g., 0.2 + 0.14; the two tenths is added to the one tenth because they are both tenths.

5-7NB I. Use a variety of strategies, including proportional reasoning, to estimate, compute, solve and explain solutions to problems involving integers, fractions, decimals and percents.

5NI 4. Round decimals to a given place value and round fractions (including mixed numbers) to the nearest half.

5NI 12. Use physical models, points of reference, and equivalent forms to add and subtract commonly used fractions with like and unlike denominators and decimals.

5NI 13. Estimate the results of computations involving whole numbers, fractions and decimals, using a variety of strategies.

Grade: Five — Measurement Standard

Measurement Standard
Students estimate and measure to a required degree of accuracy and precision by selection and using appropriate units, tools and technologies.

Mathematical Processes Benchmarks Gr. 5-7:

5-7MPB	A.	Clarify problem-solving situation and identify potential solution processes; e.g., consider different strategies and approaches to a problem, restate problem from various perspectives.
5-7MPB	B.	Apply and adapt problem-solving strategies to solve a variety of problems, including unfamiliar and non-routine problem situations.
5-7MPB	C.	Use more than one strategy to solve a problem, and recognize there are advantages associated with various methods.
5-7MPB	D.	Recognize whether an estimate or an exact solution is appropriate for a given problem situation.
5-7MPB	E.	Use deductive thinking to construct informal arguments to support reasoning and to justify solutions to problems.
5-7MPB	F.	Use inductive thinking to generalize a pattern of observations for particular cases, make conjectures, and provide supporting arguments for conjectures.
5-7MPB	G.	Relate mathematical ideas to one another and to other content areas; e.g., use area models for adding fractions, interpret graphs in reading, science and social studies.
5-7MPB	H.	Use representations to organize and communicate mathematical thinking and problem solutions.
5-7MPB	I.	Select, apply, and translate among mathematical representations to solve problems; e.g., representing a number as a fraction, decimal or percent as appropriate for a problem
5-7MPB	J.	Communicate mathematical thinking to others and analyze the mathematical thinking and strategies of others.
5-7MPB	K.	Recognize and use mathematical language and symbols when reading, writing and conversing with others.

Benchmarks Gr. 5-7:
Indicators Gr. 5:

5-7MB A. Select appropriate units to measure angles, surface area, mass and volume, using:
- U.S. customary units; e.g., degrees, square feet, pounds, and other units as appropriate;
- metric units; e.g., square meters, kilograms and other units as appropriate.

5MI	1.	Identify and select appropriate units to measure angles; i.e., degrees.
5MI	2.	Identify paths between points on a grid or coordinate plane and compare the lengths of the paths; e.g., shortest path, paths of equal length.
5MI	3.	Demonstrate and describe the differences between covering the faces (surface area) and filling the interior (volume) of three-dimensional objects.
5MI	4.	Demonstrate understanding of the differences among linear units, square units and cubic units.

5-7MB B. Convert units of length, area, volume, mass and time within the same measurement system.

5MI	5.	Make conversions within the same measurement system while performing computations.

5-7MB C. Identify appropriate tools and apply appropriate techniques for measuring angles, perimeter or circumference and area of triangles, quadrilaterals, circles and composite shapes, and surface area and volume of prisms and cylinders.

5MI 6. Use strategies to develop formulas for determining perimeter and area of triangles, rectangles and parallelograms, and volume of rectangular prisms.

5MI 7. Use benchmark angles (e.g.; 45º, 90º, 120º) to estimate the measure of angles, and use a tool to measure and draw angles.

5-7MB D. Select a tool and measure accurately to a specified level of precision.

5-7MB E. Use problem solving techniques and technology as needed to solve problems involving length, weight, perimeter, area, volume, time and temperature.

5-7MB F. Analyze and explain what happens to area and perimeter or surface area and volume when the dimensions of an object are changed.

5-7MB G. Understand and demonstrate the independence of perimeter and area for two-dimensional shapes and of surface area and volume for three-dimensional shapes.

Grade: Five — Geometry and Spatial Sense Standard

Geometry and Spatial Sense Standard
Students identify, classify, compare and analyze characteristics, properties and relationships of one-, two- and three-dimensional geometric figures and objects. Students use spatial reasoning, properties of geometric objects, and transformations to analyze mathematical situations and solve problems.

Mathematical Processes Benchmarks Gr. 5-7:

5-7MPB	A.	Clarify problem-solving situation and identify potential solution processes; e.g., consider different strategies and approaches to a problem, restate problem from various perspectives.
5-7MPB	B.	Apply and adapt problem-solving strategies to solve a variety of problems, including unfamiliar and non-routine problem situations.
5-7MPB	C.	Use more than one strategy to solve a problem, and recognize there are advantages associated with various methods.
5-7MPB	D.	Recognize whether an estimate or an exact solution is appropriate for a given problem situation.
5-7MPB	E.	Use deductive thinking to construct informal arguments to support reasoning and to justify solutions to problems.
5-7MPB	F.	Use inductive thinking to generalize a pattern of observations for particular cases, make conjectures, and provide supporting arguments for conjectures.
5-7MPB	G.	Relate mathematical ideas to one another and to other content areas; e.g., use area models for adding fractions, interpret graphs in reading, science and social studies.
5-7MPB	H.	Use representations to organize and communicate mathematical thinking and problem solutions.
5-7MPB	I.	Select, apply, and translate among mathematical representations to solve problems; e.g., representing a number as a fraction, decimal or percent as appropriate for a problem
5-7MPB	J.	Communicate mathematical thinking to others and analyze the mathematical thinking and strategies of others.
5-7MPB	K.	Recognize and use mathematical language and symbols when reading, writing and conversing with others.

Benchmarks 5-7:
Indicators Gr. 5:

5-7GSB	A.	Identify and label angle parts and the regions defined within the plane where the angle resides.		
		5GSI	2.	Use standard language to describe line, segment, ray, angle, skew, parallel and perpendicular.
5-7GSB	B.	Draw circles, and identify and determine the relationships among the radius, diameter, center and circumference.		
		5GSI	1.	Draw circles, and identify and determine relationships among the radius, diameter, center and circumference; e.g., radius is half the diameter, the ratio of the circumference of a circle to its diameter is an approximation of π.
		5GSI	3.	Label vertex, rays, interior and exterior for an angle.
		5GSI	4.	Describe and use properties of congruent figures to solve problems.
5-7GSB	C.	Specify locations and plot ordered pairs on a coordinate plane		
		5GSI	6.	Extend understanding of coordinate system to include points whose *x* or *y* values may be negative numbers.
5-7GSB	D.	Identify, describe and classify types of line pairs, angles, two-dimensional figures and three-dimensional objects using their properties.		
		5GSI	2.	Use standard language to describe line, segment, ray, angle, skew, parallel and perpendicular.

		5GSI 5. Use physical models to determine the sum of the interior angles of triangles and quadrilaterals. 5GSI 7. Understand that the measure of an angle is determined by the degree of rotation of an angle side rather than the length of either side.
5-7GSB	E.	Use proportions to express relationships among corresponding parts of similar figures.
5-7GSB	F.	Describe and use the concepts of congruence, similarity and symmetry to solve problems.
5-7GSB	G.	Describe and use properties of triangles to solve problems involving angle measures and side lengths of right triangles. 5GSI 4. Describe and use properties of congruent figures to solve problems.
5-7GSB	H.	Predict and describe results (size, position, orientation) of transformations of two-dimensional figures.
5-7GSB	I.	Identify and draw three-dimensional objects from different views (top, side, front and perspective).
		5GSI 8. Predict what three-dimensional object will result from folding a two-dimensional net, then confirm the prediction by folding the net.
5-7GSB	J.	Apply properties of equality and proportionality to solve problems involving congruent or similar figures; e.g., create a scale drawing. 5GSI 3. Label vertex, rays, interior and exterior for an angle. 5GSI 4. Describe and use properties of congruent figures to solve problems. 5GSI 5. Use physical models to determine the sum of the interior angles of triangles and quadrilaterals.

Grade: Five — Patterns, Functions and Algebra Standard

Patterns, Functions and Algebra Standard
Students use patterns, relations and functions to model, represent and analyze problem situations that involve variable quantities. Students analyze, model and solve problems using various representations such as tables, graphs and equations.

Mathematical Processes Benchmarks Gr. 5-7:

5-7MPB	A.	Clarify problem-solving situation and identify potential solution processes; e.g., consider different strategies and approaches to a problem, restate problem from various perspectives.
5-7MPB	B.	Apply and adapt problem-solving strategies to solve a variety of problems, including unfamiliar and non-routine problem situations.
5-7MPB	C.	Use more than one strategy to solve a problem, and recognize there are advantages associated with various methods.
5-7MPB	D.	Recognize whether an estimate or an exact solution is appropriate for a given problem situation.
5-7MPB	E.	Use deductive thinking to construct informal arguments to support reasoning and to justify solutions to problems.
5-7MPB	F.	Use inductive thinking to generalize a pattern of observations for particular cases, make conjectures, and provide supporting arguments for conjectures.
5-7MPB	G.	Relate mathematical ideas to one another and to other content areas; e.g., use area models for adding fractions, interpret graphs in reading, science and social studies.
5-7MPB	H.	Use representations to organize and communicate mathematical thinking and problem solutions.
5-7MPB	I.	Select, apply, and translate among mathematical representations to solve problems; e.g., representing a number as a fraction, decimal or percent as appropriate for a problem
5-7MPB	J.	Communicate mathematical thinking to others and analyze the mathematical thinking and strategies of others.
5-7MPB	K.	Recognize and use mathematical language and symbols when reading, writing and conversing with others.

Benchmarks Gr. 5-7:
Indicators Gr. 5:

5-7PFAB	A.	Describe, extend and determine the rule for patterns and relationships occurring in numeric patterns, computation, geometry, graphs and other applications.
		5PFAI 1. Justify a general rule for a pattern or a function by using physical materials, visual representations, words, tables or graphs.
		5PFAI 2. Use calculators or computers to develop patterns, and generalize them using tables and graphs.
5-7PFAB	B.	Represent, analyze and generalize a variety of patterns and functions with tables, graphs, words and symbolic rules.
5-7PFAB	C.	Use variables to create and solve equations and inequalities representing problem situations.
		5PFAI 4. Create and interpret the meaning of equations and inequalities representing problem situations.
5-7PFAB	D.	Use symbolic algebra to represent and explain mathematical relationships.
5-7PFAB	E.	Use rules and variables to describe patterns, functions and other relationships.

5PFAI 3 Use variables as unknown quantities in general rules when describing patterns and other relationships.

5-7PFAB F. Use representations, such as tables, graphs and equations, to model situations and to solve problems, especially those that involve linear relationships.
5PFAI 5. Model problems with physical materials and visual representations, and use models, graphs and tables to draw conclusions and make predictions.

5-7PFAB G. Write, simplify and evaluate algebraic expressions.
5PFAI 3 Use variables as unknown quantities in general rules when describing patterns and other relationships.

5-7PFAB H. Solve linear equations and inequalities symbolically, graphically and numerically.

5-7PFAB I. Explain how inverse operations are used to solve linear equations.

5-7PFAB J. Use formulas in problem-solving situations.

5-7PFAB K. Graph linear equations and inequalities.
5PFAI 5. Model problems with physical materials and visual representations, and use models, graphs and tables to draw conclusions and make predictions.

5-7PFAB L. Analyze functional relationships, and explain how a change in one quantity results in a change in the other.
5PFAI 6. Describe how the quantitative change in a variable affects the value of a related variable; e.g., describe how the rate of growth varies over time, based upon data in a table or graph.

5-7PFAB M. Approximate and interpret rates of change from graphical and numerical data.

Grade: Five — Data Analysis and Probability Standard

Data Analysis and Probability Standard:
Students pose questions and collect, organize, represent, interpret and analyze data to answer those questions. Students develop and evaluate inferences, predictions and arguments that are based on data.

Mathematical Processes Benchmarks Gr. 5-7:

5-7MPB	A.	Clarify problem-solving situation and identify potential solution processes; e.g., consider different strategies and approaches to a problem, restate problem from various perspectives.
5-7MPB	B.	Apply and adapt problem-solving strategies to solve a variety of problems, including unfamiliar and non-routine problem situations.
5-7MPB	C.	Use more than one strategy to solve a problem, and recognize there are advantages associated with various methods.
5-7MPB	D.	Recognize whether an estimate or an exact solution is appropriate for a given problem situation.
5-7MPB	E.	Use deductive thinking to construct informal arguments to support reasoning and to justify solutions to problems.
5-7MPB	F.	Use inductive thinking to generalize a pattern of observations for particular cases, make conjectures, and provide supporting arguments for conjectures.
5-7MPB	G.	Relate mathematical ideas to one another and to other content areas; e.g., use area models for adding fractions, interpret graphs in reading, science and social studies.
5-7MPB	H.	Use representations to organize and communicate mathematical thinking and problem solutions.
5-7MPB	I.	Select, apply, and translate among mathematical representations to solve problems; e.g., representing a number as a fraction, decimal or percent as appropriate for a problem
5-7MPB	J.	Communicate mathematical thinking to others and analyze the mathematical thinking and strategies of others.
5-7MPB	K.	Recognize and use mathematical language and symbols when reading, writing and conversing with others.

Benchmarks Gr. 5-7:
Indicators Gr. 5:

5-7DPB	A.	Read, create and use line graphs, histograms, circle graphs, box-and-whisker plots, stem-and-leaf plots, and other representations when appropriate. 5DPI 1. Read, construct and interpret frequency tables, circle graphs and line graphs.
5-7DPB	B.	Interpret data by looking for patterns and relationships draw and justify conclusions, and answer related questions.
5-7DPB	C.	Evaluate interpretations and conclusions as additional data are collected, modify conclusions and predictions, and justify new findings. 5DPI 5. Modify initial conclusions, propose and justify new interpretations and predictions as additional data are collected.
5-7DPB	D.	Compare increasingly complex displays of data, such as multiple sets of data on the same graph. 5DPI 3. Read and interpret increasingly complex displays of data, such as double bar graphs.
5-7DPB	E.	Collect, organize, display and interpret data for a specific purpose or need.

		5DPI 2.	Select and use a graph that is appropriate for the type of data to be displayed; e.g., numerical vs. categorical data, discrete vs. continuous data.
		5DPI 4.	Determine appropriate data to be collected to answer questions posed by students or teacher, collect and display data, and clearly communicate findings.
5-7DPB	F.	Determine and use the range, mean, median and mode to analyze and explain what each indicates about the data.	
		5DPI 6.	Determine and use the range, mean, median and mode, and explain what each does and does not indicate about the set of data.
5-7DPB	G.	Evaluate conjectures and predictions based upon data presented in tables and graphs, and identify misuses of statistical data and displays.	
5-7DPB	H.	Find all possible outcomes of simple experiments of problem situations, using methods such as lists, arrays and tree diagrams.	
		5DPI 7.	List and explain all possible outcomes in a given situation.
5-7DPB	I.	Describe the probability of an event using ratios, including fractional notation.	
		5DPI 8.	Identify the probability of events within a simple experiment, such as three chances out of eight.
		5DPI 9.	Use 0, 1 and ratios between 0 and 1 to represent the probability of outcomes for an event, and associate the ratio with the likelihood of the outcome.
5-7DPB	J.	Compare experimental and theoretical results for a variety of simple experiments.	
		5DPI 10.	Compare what should happen (theoretical/expected results) with what did happen (experimental/actual results) in a simple experiment.
5-7DPB	K.	Make and justify predictions based on experimental and theoretical probabilities.	
		5DPI 11.	Make predictions based on experimental and theoretical probabilities.

Grade: Six — Number, Number Sense and Operations Standard

Number, Number Sense Standard:
Students demonstrate number sense including an understanding of number systems and operations, and how they relate to one another. Students compute fluently and make reasonable estimates using paper and pencil, technology-supported and mental methods.

Mathematical Processes Benchmarks 5-7:

5-7MPB	A.	Clarify problem-solving situation and identify potential solution processes; e.g., consider different strategies and approaches to a problem, restate problem from various perspectives.
5-7MPB	B.	Apply and adapt problem-solving strategies to solve a variety of problems, including unfamiliar and non-routine problem situations.
5-7MPB	C.	Use more than one strategy to solve a problem, and recognize there are advantages associated with various methods.
5-7MPB	D.	Recognize whether an estimate or an exact solution is appropriate for a given problem situation.
5-7MPB	E.	Use deductive thinking to construct informal arguments to support reasoning and to justify solutions to problems.
5-7MPB	F.	Use inductive thinking to generalize a pattern of observations for particular cases, make conjectures, and provide supporting arguments for conjectures.
5-7MPB	G.	Relate mathematical ideas to one another and to other content areas; e.g., use area models for adding fractions, interpret graphs in reading, science and social studies.
5-7MPB	H.	Use representations to organize and communicate mathematical thinking and problem solutions.
5-7MPB	I.	Select, apply, and translate among mathematical representations to solve problems; e.g., representing a number as a fraction, decimal or percent as appropriate for a problem
5-7MPB	J.	Communicate mathematical thinking to others and analyze the mathematical thinking and strategies of others.
5-7MPB	K.	Recognize and use mathematical language and symbols when reading, writing and conversing with others.

Benchmarks/Indicators 5-7:
Indicators Gr. 6:

5-7NB A. Represent and compare numbers less than 0 through familiar applications and extending the number line.

5-7NB B. Compare, order and convert among fractions, decimals and percents.

5-7NB C. Develop meaning for percents, including percents greater than 100 and less than 1.

- 6NI 4. Describe what it means to find a specific percent of a number, using real-life examples.
- 6NI 5. Use models and pictures to relate concepts of ratio, proportion and percent, including percents less than 1 and greater than 100.

5-7NB D. Use models and pictures to relate concepts of ratio, proportion and percent.

- 6NI 3. Explain why a number is referred to as being "rational," and recognize that the expression a/b can mean *a* parts of size 1/b each, *a* divided by *b*, or the ratio of *a* to *b*.
- 6NI 5. Use models and pictures to relate concepts of ratio, proportion and percent, including percents less than 1 and greater than 100.
- 6NI 9. Give examples of how ratios are used to represent comparisons; e.g., part-to-part, part-to-whole, whole-to-part.

6NI 10. Recognize that a quotient may be larger than the dividend when the divisor is a fraction; e.g., 6 ÷ ½ = 12.

5-7NB E. Use order of operations, including use of parenthesis and exponents to solve multi-step problems, and verify and interpret the results.

6NI 6. Use the order of operations, including the use of exponents, decimals and rational numbers, to simplify numerical expressions.

5-7NB F. Apply number system properties when performing computations.

5-7NB G. Apply and explain the use of common factors, and common multiples in problem situations.

6NI 1. Decompose and recompose whole numbers using factors and exponents (e.g., 32 = 2 x 2 x 2 x 2 x 2 =2^), and explain why "squared" means second power" and "cubed" means "third power."

6NI 2. Find and use the prime factorization of composite numbers. For example:
a. Use the prime factorization to recognize the greatest common factor .
b. Use the prime factorization to recognize the least common multiple.
c. Apply the prime factorization to solve problems and explain solutions.

5-7NB H. Use and analyze the steps in standard and non-standard algorithms for computing with fractions, decimals and integers.

6NI 8. Represent multiplication and division situations involving fractions and decimals with models and visual representations; e.g., show with pattern blocks what it means to take 2 2/3 ÷ 1/6.

6NI 12. Develop and analyze algorithms for computing with fractions and decimals, and demonstrate fluency in their use.

6NI 13. Estimate reasonable solutions to problem situations involving fractions and decimals; e.g., 7/8 + 12/13 ≈ 2 an 4.23 x 5.8 ≈ 25.

5-7NB I. Use a variety of strategies, including proportional reasoning, to estimate, compute, solve and explain solutions to problems involving integers, fractions, decimals and percents.

6NI 7. Use simple expressions involving integers to represent and solve problems; e.g., if a running back loses 15 yards on the first carry but gains 8 yards on the second carry, what is the net gain/loss?

6NI 11. Perform fraction and decimal computations and justify their solutions; e.g., using manipulative, diagrams, mathematical reasoning.

6NI 14. Use proportional reasoning, ratios and percents to represent problem situations and determine the reasonableness of solutions.

6NI 15. Determine the percent of a number and solve related problems; e.g., find the percent markdown if the original price was $140, and the sale price is $100.

Grade: Six — Measurement Standard

Measurement Standard
Students estimate and measure to a required degree of accuracy and precision by selection and using appropriate units, tools and technologies.

Mathematical Processes Benchmarks 5-7:

5-7MPB	A.	Clarify problem-solving situation and identify potential solution processes; e.g., consider different strategies and approaches to a problem, restate problem from various perspectives.
5-7MPB	B.	Apply and adapt problem-solving strategies to solve a variety of problems, including unfamiliar and non-routine problem situations.
5-7MPB	C.	Use more than one strategy to solve a problem, and recognize there are advantages associated with various methods.
5-7MPB	D.	Recognize whether an estimate or an exact solution is appropriate for a given problem situation.
5-7MPB	E.	Use deductive thinking to construct informal arguments to support reasoning and to justify solutions to problems.
5-7MPB	F.	Use inductive thinking to generalize a pattern of observations for particular cases, make conjectures, and provide supporting arguments for conjectures.
5-7MPB	G.	Relate mathematical ideas to one another and to other content areas; e.g., use area models for adding fractions, interpret graphs in reading, science and social studies.
5-7MPB	H.	Use representations to organize and communicate mathematical thinking and problem solutions.
5-7MPB	I.	Select, apply, and translate among mathematical representations to solve problems; e.g., representing a number as a fraction, decimal or percent as appropriate for a problem
5-7MPB	J.	Communicate mathematical thinking to others and analyze the mathematical thinking and strategies of others.
5-7MPB	K.	Recognize and use mathematical language and symbols when reading, writing and conversing with others.

Benchmarks Gr. 5-7:
Indicators Gr. 6:

5-7MB A. Select appropriate units to measure angles, surface area, mass and volume, using:
- U.S. customary units; e.g., degrees, square feet, pounds, and other units as appropriate;
- metric units; e.g., square meters, kilograms and other units as appropriate.

5-7MB B. Convert units of length, area, volume, mass and time within the same measurement system.

5-7MB C. Identify appropriate tools and apply appropriate techniques for measuring angles, perimeter or circumference and area of triangles, quadrilaterals, circles and composite shapes, and surface area and volume of prisms and cylinders.

6MI 2. Use strategies to develop formulas for finding circumference and area of circles, and to determine the area of sectors; e.g., ½ circle, 2/3 circle, 1/3 circle, ¼ circle.

6MI 3. Estimate perimeter or circumference and area for circles, triangles and quadrilaterals, and surface area and volume for prisms and cylinders by:
a. estimating lengths using string or links, areas using tiles or grid, and volumes using cubes
b. measuring attributes (diameter, side lengths, or heights) and using established formulas for circles, triangles, rectangles, parallelograms and rectangular prisms.

5-7MB D. Select a tool and measure accurately to a specified level of precision.

5-7MB E. Use problem solving techniques and technology as needed to solve problems involving length, weight, perimeter, area, volume, time and temperature.

6MI 4. Determine which measure (perimeter, area, surface area, volume) matches the context for a problem situation; e.g., perimeter is the context for fencing a garden, surface area is the context for painting a room.

5-7MB F. Analyze and explain what happens to area and perimeter or surface area and volume when the dimensions of an object are changed.

6MI 6. Describe what happens to the perimeter and area of a two-dimensional shape when the measurements of the shape are changed; e.g. length of sides are doubled.

5-7MB G. Understand and demonstrate the independence of perimeter and area for two-dimensional shapes and of surface area and volume for three-dimensional shapes.

6MI 1. Understand and describe the difference between surface area and volume.

6MI 5. Understand the difference between perimeter and area, and demonstrate that two shapes may have the same perimeter, but different areas or may have the same area, but different perimeters.

Grade: Six — Geometry and Spatial Sense Standard

Geometry and Spatial Sense Standard
Students identify, classify, compare and analyze characteristics, properties and relationships of one-, two- and three-dimensional geometric figures and objects. Students use spatial reasoning, properties of geometric objects, and transformations to analyze mathematical situations and solve problems.

Mathematical Processes Benchmarks 5-7:

5-7MPB	A.	Clarify problem-solving situation and identify potential solution processes; e.g., consider different strategies and approaches to a problem, restate problem from various perspectives.
5-7MPB	B.	Apply and adapt problem-solving strategies to solve a variety of problems, including unfamiliar and non-routine problem situations.
5-7MPB	C.	Use more than one strategy to solve a problem, and recognize there are advantages associated with various methods.
5-7MPB	D.	Recognize whether an estimate or an exact solution is appropriate for a given problem situation.
5-7MPB	E.	Use deductive thinking to construct informal arguments to support reasoning and to justify solutions to problems.
5-7MPB	F.	Use inductive thinking to generalize a pattern of observations for particular cases, make conjectures, and provide supporting arguments for conjectures.
5-7MPB	G.	Relate mathematical ideas to one another and to other content areas; e.g., use area models for adding fractions, interpret graphs in reading, science and social studies.
5-7MPB	H.	Use representations to organize and communicate mathematical thinking and problem solutions.
5-7MPB	I.	Select, apply, and translate among mathematical representations to solve problems; e.g., representing a number as a fraction, decimal or percent as appropriate for a problem
5-7MPB	J.	Communicate mathematical thinking to others and analyze the mathematical thinking and strategies of others.
5-7MPB	K.	Recognize and use mathematical language and symbols when reading, writing and conversing with others.

Benchmarks Gr. 5-7:
Indicators Gr. 6:

5-7GSB	A.			Identify and label angle parts and the regions defined within the plane where the angle resides.
5-7GSB	B.			Draw circles, and identify and determine the relationships among the radius, diameter, center and circumference.
5-7GSB	C.			Specify locations and plot ordered pairs on a coordinate plane.
5-7GSB	D.			Identify, describe and classify types of line pairs, angles, two-dimensional figures and three-dimensional objects using their properties.
		6GSI	1.	Classify and describe two-dimensional and three-dimensional geometric figures and objects by using their properties; e.g., interior angle measures, perpendicular/parallel sides, congruent angles/sides.
		6GSI	2.	Use standard language to define geometric vocabulary: vertex, face, altitude, diagonal, isosceles, equilateral, acute, obtuse and other vocabulary as appropriate.
		6GSI	4.	Identify and define relationships between planes; i.e., parallel, perpendicular and intersecting.

5-7GSB	E.	Use proportions to express relationships among corresponding parts of similar figures.
5-7GSB	F.	Describe and use the concepts of congruence, similarity and symmetry to solve problems.
5-7GSB	G.	Describe and use properties of triangles to solve problems involving angle measures and side lengths of right triangles. 6GSI 3. Use multiple classification criteria to classify triangles; e.g., right scalene triangle.
5-7GSB	H.	Predict and describe results (size, position, orientation) of transformations of two-dimensional figures. 6GSI 5. Predict and describe sizes, positions and orientations of two-dimensional shapes after transformations such as reflections, rotations, translations and dilations.
5-7GSB	I.	Identify and draw three-dimensional objects from different views (top, side, front and perspective).
5-7GSB	J.	Apply properties of equality and proportionality to solve problems involving congruent or similar figures; e.g., create a scale drawing. 6GSI 6. Draw similar figures that model proportional relationships; e.g., model similar figures with a 1 to 2 relationship by sketching two of the same figure, one with corresponding sides twice the length of the other. 6GSI 7. Build three-dimensional objects with cubes, and sketch the two-dimensional representations of each side; i.e., projection sets.

Grade: Six — Patterns, Functions and Algebra Standard

Patterns, Functions and Algebra Standard
Students use patterns, relations and functions to model, represent and analyze problem situations that involve variable quantities. Students analyze, model and solve problems using various representations such as tables, graphs and equations.

Mathematical Processes Benchmarks 5-7:

5-7MPB	A.	Clarify problem-solving situation and identify potential solution processes; e.g., consider different strategies and approaches to a problem, restate problem from various perspectives.
5-7MPB	B.	Apply and adapt problem-solving strategies to solve a variety of problems, including unfamiliar and non-routine problem situations.
5-7MPB	C.	Use more than one strategy to solve a problem, and recognize there are advantages associated with various methods.
5-7MPB	D.	Recognize whether an estimate or an exact solution is appropriate for a given problem situation.
5-7MPB	E.	Use deductive thinking to construct informal arguments to support reasoning and to justify solutions to problems.
5-7MPB	F.	Use inductive thinking to generalize a pattern of observations for particular cases, make conjectures, and provide supporting arguments for conjectures.
5-7MPB	G.	Relate mathematical ideas to one another and to other content areas; e.g., use area models for adding fractions, interpret graphs in reading, science and social studies.
5-7MPB	H.	Use representations to organize and communicate mathematical thinking and problem solutions.
5-7MPB	I.	Select, apply, and translate among mathematical representations to solve problems; e.g., representing a number as a fraction, decimal or percent as appropriate for a problem
5-7MPB	J.	Communicate mathematical thinking to others and analyze the mathematical thinking and strategies of others.
5-7MPB	K.	Recognize and use mathematical language and symbols when reading, writing and conversing with others.

Benchmarks Gr. 5-7:
Indicators Gr. 6:

5-7PFAB	A.	Describe, extend and determine the rule for patterns and relationships occurring in numeric patterns, computation, geometry, graphs and other applications. 6PFAI 1. Represent and analyze patterns, rules and functions, using physical materials, tables and graphs. 6PFAI 2. Use words and symbols to describe numerical and geometric patterns, rules and functions. (Find the nth term in a sequence of numbers involving one operation or power).
5-7PFAB	B.	Represent, analyze and generalize a variety of patterns and functions with tables, graphs, words and symbolic rules.
5-7PFAB	C.	Use variables to create and solve equations and inequalities representing problem situations. 6PFAI 5. Produce and interpret graphs that represent the relationship between two variables. 6PFAI 6. Evaluate simple expressions by replacing variables with given values, and use formulas in problem-solving situations.
5-7PFAB	D.	Use symbolic algebra to represent and explain mathematical relationships.

6PFAI 3. Recognize and generate equivalent forms of algebraic expressions, and explain how the commutative, associative and distributive properties can be used to generate equivalent forms; e.g., perimeter as $2(l + w)$ or $2l + 2w$.

5-7PFAB E. Use rules and variables to describe patterns, functions and other relationships.
6PFAI 2. Use words and symbols to describe numerical and geometric patterns, rules and functions. (Find the nth term in a sequence of numbers involving one operation or power).

5-7PFAB F. Use representations, such as tables, graphs and equations, to model situations and to solve problems, especially those that involve linear relationships.

5-7PFAB G. Write, simplify and evaluate algebraic expressions.
6PFAI 6. Evaluate simple expressions by replacing variables with given values, and use formulas in problem-solving situations.

5-7PFAB H. Solve linear equations and inequalities symbolically, graphically and numerically.
6PFAI 4. Solve simple linear equations and inequalities using physical models, paper and pencil, tables and graphs.

5-7PFAB I. Explain how inverse operations are used to solve linear equations.

5-7PFAB J. Use formulas in problem-solving situations.
6PFAI 6. Evaluate simple expressions by replacing variables with given values, and use formulas in problem-solving situations.

5-7PFAB K. Graph linear equations and inequalities.
6PFAI 4. Solve simple linear equations and inequalities using physical models, paper and pencil, tables and graphs.
6PFAI 5. Produce and interpret graphs that represent the relationship between two variables.

5-7PFAB L. Analyze functional relationships, and explain how a change in one quantity results in a change in the other.
6PFAI 7. Identify and describe situations with constant or varying rates of change, and compare them.

5-7PFAB M. Approximate and interpret rates of change from graphical and numerical data.
6PFAI 8. Use technology to analyze change; e.g., use computer applications or graphing calculators to display and interpret rate of change.

Grade: Six — Data Analysis and Probability Standard

Data Analysis and Probability Standard:
Students pose questions and collect, organize, represent, interpret and analyze data to answer those questions. Students develop and evaluate inferences, predictions and arguments that are based on data.

Mathematical Processes Benchmarks 5-7:

5-7MPB	A.	Clarify problem-solving situation and identify potential solution processes; e.g., consider different strategies and approaches to a problem, restate problem from various perspectives.
5-7MPB	B.	Apply and adapt problem-solving strategies to solve a variety of problems, including unfamiliar and non-routine problem situations.
5-7MPB	C.	Use more than one strategy to solve a problem, and recognize there are advantages associated with various methods.
5-7MPB	D.	Recognize whether an estimate or an exact solution is appropriate for a given problem situation.
5-7MPB	E.	Use deductive thinking to construct informal arguments to support reasoning and to justify solutions to problems.
5-7MPB	F.	Use inductive thinking to generalize a pattern of observations for particular cases, make conjectures, and provide supporting arguments for conjectures.
5-7MPB	G.	Relate mathematical ideas to one another and to other content areas; e.g., use area models for adding fractions, interpret graphs in reading, science and social studies.
5-7MPB	H.	Use representations to organize and communicate mathematical thinking and problem solutions.
5-7MPB	I.	Select, apply, and translate among mathematical representations to solve problems; e.g., representing a number as a fraction, decimal or percent as appropriate for a problem
5-7MPB	J.	Communicate mathematical thinking to others and analyze the mathematical thinking and strategies of others.
5-7MPB	K.	Recognize and use mathematical language and symbols when reading, writing and conversing with others.

Benchmarks Gr. 5-7:
Indicators Gr. 6

5-7DPB	A.	Read, create and use line graphs, histograms, circle graphs, box-and-whisker plots, stem-and-leaf plots, and other representations when appropriate.
		6DPI 1. Read, construct and interpret line graphs, circle graphs and histograms.
5-7DPB	B.	Interpret data by looking for patterns and relationships draw and justify conclusions, and answer related questions.
		6DPI 5. Describe the frequency distribution of a set of data, as shown in a histogram or frequency table, by general appearance or shape; e.g., number of modes, middle of data, level of symmetry, outliers.
5-7DPB	C.	Evaluate interpretations and conclusions as additional data are collected, modify conclusions and predictions, and justify new findings.
5-7DPB	D.	Compare increasingly complex displays of data, such as multiple sets of data on the same graph.
		6DPI 3. Compare representations of the same data in different types of graphs, such as a bar graph and circle graph.

5-7DPB	E.	Collect, organize, display and interpret data for a specific purpose or need. 6DPI 2. Select, create and use graphical representations that are appropriate for the type of data collected.
5-7DPB	F.	Determine and use the range, mean, median and mode to analyze and explain what each indicates about the data. 6DPI 4. Understand the different information provided by measures of center (mean, mode and median) and measures of spread (range).
5-7DPB	G.	Evaluate conjectures and predictions based upon data presented in tables and graphs, and identify misuses of statistical data and displays. 6DPI 6. Make logical inferences from statistical data.
5-7DPB	H.	Find all possible outcomes of simple experiments of problem situations, using methods such as lists, arrays and tree diagrams.
5-7DPB	I.	Describe the probability of an event using ratios, including fractional notation.
5-7DPB	J.	Compare experimental and theoretical results for a variety of simple experiments.
5-7DPB	K.	Make and justify predictions based on experimental and theoretical probabilities. 6DPI 7. Design an experiment to test a theoretical probability and explain how the results may vary.

Grade: Seven — Number, Number Sense and Operations Standard

Number, Number Sense Standard:
Students demonstrate number sense including an understanding of number systems and operations, and how they relate to one another. Students compute fluently and make reasonable estimates using paper and pencil, technology-supported and mental methods.

Mathematical Processes Benchmarks Gr. 5-7:

5-7MPB	A.	Clarify problem-solving situation and identify potential solution processes; e.g., consider different strategies and approaches to a problem, restate problem from various perspectives.
5-7MPB	B.	Apply and adapt problem-solving strategies to solve a variety of problems, including unfamiliar and non-routine problem situations.
5-7MPB	C.	Use more than one strategy to solve a problem, and recognize there are advantages associated with various methods.
5-7MPB	D.	Recognize whether an estimate or an exact solution is appropriate for a given problem situation.
5-7MPB	E.	Use deductive thinking to construct informal arguments to support reasoning and to justify solutions to problems.
5-7MPB	F.	Use inductive thinking to generalize a pattern of observations for particular cases, make conjectures, and provide supporting arguments for conjectures.
5-7MPB	G.	Relate mathematical ideas to one another and to other content areas; e.g., use area models for adding fractions, interpret graphs in reading, science and social studies.
5-7MPB	H.	Use representations to organize and communicate mathematical thinking and problem solutions.
5-7MPB	I.	Select, apply, and translate among mathematical representations to solve problems; e.g., representing a number as a fraction, decimal or percent as appropriate for a problem
5-7MPB	J.	Communicate mathematical thinking to others and analyze the mathematical thinking and strategies of others.
5-7MPB	K.	Recognize and use mathematical language and symbols when reading, writing and conversing with others.

Benchmarks Gr.5-7:
Indicators Gr. 7:

5-7NB	A.	Represent and compare numbers less than 0 through familiar applications and extending the number line.
	7NI 2.	Explain the meaning of exponents that are negative or 0.
5-7NB	B.	Compare, order and convert among fractions, decimals and percents.
5-7NB	C.	Develop meaning for percents, including percents greater than 100 and less than 1.
5-7NB	D.	Use models and pictures to relate concepts of ratio, proportion and percent.
5-7NB	E.	Use order of operations, including use of parenthesis and exponents to solve multi-step problems, and verify and interpret the results.
	7NI 1.	Demonstrate an understanding of place value using powers of 10 and write large numbers in scientific notation.
	7NI 2.	Explain the meaning of exponents that are negative or 0.
	7NI 4.	Use order of operations and properties to simplify numerical expressions involving integers, fractions and decimals.

5-7NB	F.	Apply number system properties when performing computations.		
		7NI	3.	Describe differences between rational and irrational numbers; e.g., use technology to show that some numbers (rational) can be expressed as terminating or repeating decimals and others (irrational) as non-terminating and non-repeating decimals.
5-7NB	G.	Apply and explain the use of common factors, and common multiples in problem situations.		
		7NI	9.	Represent and solve problem situations that can be modeled by and solved using concepts of absolute value, exponents and square roots (for perfect squares).
5-7NB	H.	Use and analyze the steps in standard and non-standard algorithms for computing with fractions, decimals and integers.		
		7NI	5.	Explain the meaning and effect of adding, subtracting, multiplying and dividing integers; e.g., how adding two integers can result in a lesser value.
		7NI	8.	Develop and analyze algorithms for computing with percents and integers, and demonstrate fluency in their use.
5-7NB	I.	Use a variety of strategies, including proportional reasoning, to estimate, compute, solve and explain solutions to problems involving integers, fractions, decimals and percents.		
		7NI	6.	Simplify numerical expressions involving integers and use integers to solve real-life problems.
		7NI	7.	Solve problems using the appropriate form of a rational number (fraction, decimal or percent).
		7NI	9.	Represent and solve problem situations that can be modeled by and solved using concepts of absolute value, exponents and square roots (for perfect squares).

Grade: Seven — Measurement Standard

Measurement Standard
Students estimate and measure to a required degree of accuracy and precision by selection and using appropriate units, tools and technologies.

Mathematical Processes Benchmarks Gr. 5-7:

5-7MPB	A.	Clarify problem-solving situation and identify potential solution processes; e.g., consider different strategies and approaches to a problem, restate problem from various perspectives.
5-7MPB	B.	Apply and adapt problem-solving strategies to solve a variety of problems, including unfamiliar and non-routine problem situations.
5-7MPB	C.	Use more than one strategy to solve a problem, and recognize there are advantages associated with various methods.
5-7MPB	D.	Recognize whether an estimate or an exact solution is appropriate for a given problem situation.
5-7MPB	E.	Use deductive thinking to construct informal arguments to support reasoning and to justify solutions to problems.
5-7MPB	F.	Use inductive thinking to generalize a pattern of observations for particular cases, make conjectures, and provide supporting arguments for conjectures.
5-7MPB	G.	Relate mathematical ideas to one another and to other content areas; e.g., use area models for adding fractions, interpret graphs in reading, science and social studies.
5-7MPB	H.	Use representations to organize and communicate mathematical thinking and problem solutions.
5-7MPB	I.	Select, apply, and translate among mathematical representations to solve problems; e.g., representing a number as a fraction, decimal or percent as appropriate for a problem
5-7MPB	J.	Communicate mathematical thinking to others and analyze the mathematical thinking and strategies of others.
5-7MPB	K.	Recognize and use mathematical language and symbols when reading, writing and conversing with others.

Benchmark Gr. 5 - 7:
Indicators Gr. 7:

5-7MB A. Select appropriate units to measure angles, surface area, mass and volume, using:
- U.S. customary units; e.g., degrees, square feet, pounds, and other units as appropriate;
- metric units; e.g., square meters, kilograms and other units as appropriate.

7MI 1. Select appropriate units for measuring derived measurements; e.g., miles per hour, revolutions per minute.

5-7MB B. Convert units of length, area, volume, mass and time within the same measurement system.

7MI 2. Convert units of area and volume within the same measurement system using proportional reasoning and a reference table when appropriate; e.g., square feet to square yards, cubic meters to cubic centimeters.

5-7MB C. Identify appropriate tools and apply appropriate techniques for measuring angles, perimeter or circumference and area of triangles, quadrilaterals, circles and composite shapes, and surface area and volume of prisms and cylinders.

7MI 6. Use strategies to develop formulas for finding area of trapezoids and volume of cylinders, prisms and spheres.

7MI 7. Develop strategies to find the area of composite shapes using the areas of triangles, parallelograms, circles and sectors.

5-7MB	D.	Select a tool and measure accurately to a specified level of precision.	
		7MI 3.	Estimate a measurement to a greater degree of precision than the tool provides.
5-7MB	E.	Use problem solving techniques and technology as needed to solve problems involving length, weight, perimeter, area, volume, time and temperature.	
		7MI 4.	Solve problems involving proportional relationships and scale factors; e.g., scale models that require unit conversions within the same measurement system.
		7MI 5.	Analyze problem situations involving measurement concepts, select appropriate strategies, and use an organized approach to solve narrative and increasingly complex problems.
5-7MB	F.	Analyze and explain what happens to area and perimeter or surface area and volume when the dimensions of an object are changed.	
		7MI 9.	Describe what happens to the surface area and volume of a three-dimensional object when the measurements of the object are changed; e.g., length of sides are doubled.
5-7MB	G.	Understand and demonstrate the independence of perimeter and area for two-dimensional shapes and of surface area and volume for three-dimensional shapes.	
		7MI 8.	Understand the difference between surface area and volume and demonstrate that two objects may have the same surface area, but different volumes or may have the same volume, but different surface areas.

Grade: Seven — Geometry and Spatial Sense Standard

Geometry and Spatial Sense Standard
Students identify, classify, compare and analyze characteristics, properties and relationships of one-, two- and three-dimensional geometric figures and objects. Students use spatial reasoning, properties of geometric objects, and transformations to analyze mathematical situations and solve problems.

Mathematical Processes Benchmarks Gr. 5-7:

5-7MPB	A.	Clarify problem-solving situation and identify potential solution processes; e.g., consider different strategies and approaches to a problem, restate problem from various perspectives.
5-7MPB	B.	Apply and adapt problem-solving strategies to solve a variety of problems, including unfamiliar and non-routine problem situations.
5-7MPB	C.	Use more than one strategy to solve a problem, and recognize there are advantages associated with various methods.
5-7MPB	D.	Recognize whether an estimate or an exact solution is appropriate for a given problem situation.
5-7MPB	E.	Use deductive thinking to construct informal arguments to support reasoning and to justify solutions to problems.
5-7MPB	F.	Use inductive thinking to generalize a pattern of observations for particular cases, make conjectures, and provide supporting arguments for conjectures.
5-7MPB	G.	Relate mathematical ideas to one another and to other content areas; e.g., use area models for adding fractions, interpret graphs in reading, science and social studies.
5-7MPB	H.	Use representations to organize and communicate mathematical thinking and problem solutions.
5-7MPB	I.	Select, apply, and translate among mathematical representations to solve problems; e.g., representing a number as a fraction, decimal or percent as appropriate for a problem
5-7MPB	J.	Communicate mathematical thinking to others and analyze the mathematical thinking and strategies of others.
5-7MPB	K.	Recognize and use mathematical language and symbols when reading, writing and conversing with others.

Benchmarks Gr. 5-7:
Indicators Gr. 7:

5-7GSB	A.	Identify and label angle parts and the regions defined within the plane where the angle resides.
5-7GSB	B.	Draw circles, and identify and determine the relationships among the radius, diameter, center and circumference.
5-7GSB	C.	Specify locations and plot ordered pairs on a coordinate plane.
5-7GSB	D.	Identify, describe and classify types of line pairs, angles, two-dimensional figures and three-dimensional objects using their properties.

7GSI 2. Determine sufficient (not necessarily minimal) properties that define a specific two-dimensional figure or three-dimensional object. For example:
a. Determine when one set of figures is a subset of another; e.g., all squares are rectangles.
b. Develop a set of properties that eliminate all but the desired figure; e.g., only squares are quadrilaterals with all sides congruent and all angles congruent.

5-7GSB	E.	Use proportions to express relationships among corresponding parts of similar figures.		
		7GSI	1.	Use proportional reasoning to describe and express relationships between parts and attributes of similar and congruent figures.
		7GSI	6.	Determine and use scale factors for similar figures to solve problems using proportional reasoning.
5-7GSB	F.	Describe and use the concepts of congruence, similarity and symmetry to solve problems.		
		7GSI	4.	Determine necessary conditions for congruence of triangles.
		7GSI	7.	Identify the line and rotation symmetries of two-dimensional figures to solve problems.
5-7GSB	G.	Describe and use properties of triangles to solve problems involving angle measures and side lengths of right triangles.		
		7GSI	3.	Use and demonstrate understanding of the properties of triangles. For example: a. Use Pythagorean Theorem to solve problems involving right triangles. b. Use triangle angle sum relationships to solve problems
		7GSI	5.	Apply properties of congruent or similar triangles to solve problems involving missing lengths and angle measures.
5-7GSB	H.	Predict and describe results (size, position, orientation) of transformations of two-dimensional figures.		
		7GSI	8.	Perform translations, reflections, rotations and dilations of two-dimensional figures using a variety of methods (paper folding, tracing, graph paper).
5-7GSB	I.	Identify and draw three-dimensional objects from different views (top, side, front and perspective).		
		7GSI	9.	Draw representations of three-dimensional geometric objects from different views.
5-7GSB	J.	Apply properties of equality and proportionality to solve problems involving congruent or similar figures; e.g., create a scale drawing.		
		7GSI	1.	Use proportional reasoning to describe and express relationships between parts and attributes of similar and congruent figures.
		7GSI	3.	Use and demonstrate understanding of the properties of triangles. For example: a. Use Pythagorean Theorem to solve problems involving right triangles. b. Use triangle angle sum relationships to solve problems
		7GSI	6.	Determine and use scale factors for similar figures to solve problems using proportional reasoning.

Grade: Seven — Patterns, Functions and Algebra Standard

Patterns, Functions and Algebra Standard
Students use patterns, relations and functions to model, represent and analyze problem situations that involve variable quantities. Students analyze, model and solve problems using various representations such as tables, graphs and equations.

Mathematical Processes Benchmarks Gr. 5-7:

5-7MPB	A.	Clarify problem-solving situation and identify potential solution processes; e.g., consider different strategies and approaches to a problem, restate problem from various perspectives.
5-7MPB	B.	Apply and adapt problem-solving strategies to solve a variety of problems, including unfamiliar and non-routine problem situations.
5-7MPB	C.	Use more than one strategy to solve a problem, and recognize there are advantages associated with various methods.
5-7MPB	D.	Recognize whether an estimate or an exact solution is appropriate for a given problem situation.
5-7MPB	E.	Use deductive thinking to construct informal arguments to support reasoning and to justify solutions to problems.
5-7MPB	F.	Use inductive thinking to generalize a pattern of observations for particular cases, make conjectures, and provide supporting arguments for conjectures.
5-7MPB	G.	Relate mathematical ideas to one another and to other content areas; e.g., use area models for adding fractions, interpret graphs in reading, science and social studies.
5-7MPB	H.	Use representations to organize and communicate mathematical thinking and problem solutions.
5-7MPB	I.	Select, apply, and translate among mathematical representations to solve problems; e.g., representing a number as a fraction, decimal or percent as appropriate for a problem
5-7MPB	J.	Communicate mathematical thinking to others and analyze the mathematical thinking and strategies of others.
5-7MPB	K.	Recognize and use mathematical language and symbols when reading, writing and conversing with others.

Benchmarks Gr. 5-7:
Indicators Gr. 7:

5-7PFAB	A.	Describe, extend and determine the rule for patterns and relationships occurring in numeric patterns, computation, geometry, graphs and other applications.
5-7PFAB	B.	Represent, analyze and generalize a variety of patterns and functions with tables, graphs, words and symbolic rules.
		7PFAI 1. Represent and analyze patterns, rules and functions with words, tables, graphs and simple variable expressions.
		7PFAI 2. Generalize patterns by describing in words how to find the next term.
5-7PFAB	C.	Use variables to create and solve equations and inequalities representing problem situations.
5-7PFAB	D.	Use symbolic algebra to represent and explain mathematical relationships.
		7PFAI 9. Recognize a variety of uses for variables; e.g., placeholder for an unknown quantity in an equation, generalization for a pattern, formula.

5-7PFAB	E.	Use rules and variables to describe patterns, functions and other relationships. 7PFAI 3. Recognize and explain when numerical patterns are linear or nonlinear progressions; e.g., 1,3,5,7... is linear and 1,3,4,8,16... is nonlinear.
5-7PFAB	F.	Use representations, such as tables, graphs and equations, to model situations and to solve problems, especially those that involve linear Relationships. 7PFAI 5. Represent linear equations by plotting points in the coordinate plane. 7PAFI 6. Represent inequalities on a number line or a coordinate lane.
5-7PFAB	G.	Write, simplify and evaluate algebraic expressions. 7PFAI 1. Represent and analyze patterns, rules and functions with words, tables, graphs and simple variable expressions. 7PFAI 7. Justify that two forms of an algebraic expression are equivalent, and recognize when an expression is simplified; e.g., $4m = m + m + m + m$ or $a \cdot 5 + 4 = 5a + 4$.
5-7PFAB	H.	Solve linear equations and inequalities symbolically, graphically and numerically. 7PFAI 4. Create visual representations of equation-solving processes that model the use of inverse operations.
5-7PFAB	I.	Explain how inverse operations are used to solve linear equations. 7PFAI 4. Create visual representations of equation-solving processes that model the use of inverse operations.
5-7PFAB	J.	Use formulas in problem-solving situations. 7PFAI 8. Use formulas in problem-solving situations.
5-7PFAB	K.	Graph linear equations and inequalities. 7PFAI 5. Represent linear equations by plotting points in the coordinate plane. 7PAFI 6. Represent inequalities on a number line or a coordinate lane.
5-7PFAB	L.	Analyze functional relationships, and explain how a change in one quantity results in a change in the other. 7PFAI 10. Analyze linear and simple nonlinear relationships to explain how a change in one variable results in the change of another.
5-7PFAB	M.	Approximate and interpret rates of change from graphical and numerical data. 7PFAI 11. Use graphing calculators or computers to analyze change; e.g., distance-time relationships.

Grade: Seven — Data Analysis and Probability Standard

Data Analysis and Probability Standard:
Students pose questions and collect, organize, represent, interpret and analyze data to answer those questions. Students develop and evaluate inferences, predictions and arguments that are based on data.

Mathematical Processes Benchmarks Gr. 5-7:

5-7MPB	A.	Clarify problem-solving situation and identify potential solution processes; e.g., consider different strategies and approaches to a problem, restate problem from various perspectives.
5-7MPB	B.	Apply and adapt problem-solving strategies to solve a variety of problems, including unfamiliar and non-routine problem situations.
5-7MPB	C.	Use more than one strategy to solve a problem, and recognize there are advantages associated with various methods.
5-7MPB	D.	Recognize whether an estimate or an exact solution is appropriate for a given problem situation.
5-7MPB	E.	Use deductive thinking to construct informal arguments to support reasoning and to justify solutions to problems.
5-7MPB	F.	Use inductive thinking to generalize a pattern of observations for particular cases, make conjectures, and provide supporting arguments for conjectures.
5-7MPB	G.	Relate mathematical ideas to one another and to other content areas; e.g., use area models for adding fractions, interpret graphs in reading, science and social studies.
5-7MPB	H.	Use representations to organize and communicate mathematical thinking and problem solutions.
5-7MPB	I.	Select, apply, and translate among mathematical representations to solve problems; e.g., representing a number as a fraction, decimal or percent as appropriate for a problem
5-7MPB	J.	Communicate mathematical thinking to others and analyze the mathematical thinking and strategies of others.
5-7MPB	K.	Recognize and use mathematical language and symbols when reading, writing and conversing with others.

Benchmarks Gr. 5-7:
Indicators Gr. 7:

5-7DPB	A.	Read, create and use line graphs, histograms, circle graphs, box-and-whisker plots, stem-and-leaf plots, and other representations when appropriate. 7DPI 1. Read, create and interpret box-and-whisker plots, stem-and-leaf plots, and other types of graphs, when appropriate.
5-7DPB	B.	Interpret data by looking for patterns and relationships draw and justify conclusions, and answer related questions. 7DPI 4. Construct opposing arguments based on analysis of the same data, using different graphical representations.
5-7DPB	C.	Evaluate interpretations and conclusions as additional data are collected, modify conclusions and predictions, and justify new findings.
5-7DPB	D.	Compare increasingly complex displays of data, such as multiple sets of data on the same graph. 7DPI 5. Compare data from two or more samples to determine how sample selection can influence results.

5-7DPB	E.	Collect, organize, display and interpret data for a specific purpose or need. 7DPI 2. Analyze how decisions about graphing affect the graphical representation; e.g., scale, size of classes in a histogram, number of categories in a circle graph.
5-7DPB	F.	Determine and use the range, mean, median and mode to analyze and explain what each indicates about the data. 7DPI 3. Analyze a set of data by using and comparing combinations of measures of center (mean, mode, median) and measures of spread (range, quartile, interquartile range), and describe how the inclusion or exclusion of outliers affects those measures.
5-7DPB	G.	Evaluate conjectures and predictions based upon data presented in tables and graphs, and identify misuses of statistical data and displays. 7DPI 2. Analyze how decisions about graphing affect the graphical representation; e.g., scale, size of classes in a histogram, number of categories in a circle graph. 7DPI 6. Identify misuses of statistical data in articles, advertisements, and other media.
5-7DPB	H.	Find all possible outcomes of simple experiments of problem situations, using methods such as lists, arrays and tree diagrams.
5-7DPB	I.	Describe the probability of an event using ratios, including fractional notation. 7DPI 7. Compute probabilities of compound events; e.g., multiple coin tosses or multiple rolls of number cubes, using such methods organized lists, tree diagrams and area models.
5-7DPB	J.	Compare experimental and theoretical results for a variety of simple experiments.
5-7DPB	K.	Make and justify predictions based on experimental and theoretical probabilities. 7DPI 8. Make predictions based on theoretical probabilities, design and conduct an experiment to test the predictions, compare actual results to predicted result, and explain differences.

RESEARCH BASE: NUMBERS AND OPERATIONS
GRADES 8 - 10

INTRODUCTION:

Students in grades 8 – 10 should see number and operations from a more global perspective. Their understanding of number is the foundation for their understanding of algebra, the core of all mathematics. High school students should understand more fully the concept of a number system, how different number systems are related, and whether the properties of one system hold in another. Students will develop an increased ability to estimate the results of an arithmetic computation and judge the reasonableness of results obtained through technology.

Students in high school will understand the meaning of exponents and how to apply their properties in computations. The real number system will be explored for its use in matrices and vectors. High school students will relate complex numbers to problems for which there are no real solutions. Students will apply these concepts in a variety of problem-solving situations.

RESEARCH BASE:

Regardless of the particular method used, students should be able to explain their method, understand that many methods exist, and see the usefulness of methods that are efficient, accurate and general. Students also need to be able to estimate and judge the reasonableness of results. Computational fluency should develop in tandem with understanding of the role of meaning of arithmetic operations in number systems (Hiebert, et al., 1997; Thornton, 1990).

8-10 Number and Number Sense
ESSENTIAL FOCUS:

1. Students should know how to use scientific notation and exponents to represent large and small numbers (8^{th}).
2. Students should know how to solve problems using proportional relationships (8^{th}).
3. Students should know how to estimate, compute, and solve problems that include rational numbers (8^{th}).
4. Students should know how to use matrices (9^{th}).
5. Students should know how to compute fluently with real numbers including powers and roots (9^{th}).
6. Students should know how to represent vectors (10^{th}).

RESEARCH BASE: MEASUREMENT
GRADES 8 -10

INTRODUCTION: Opportunities to use and understand measurement arise naturally during high school in various disciplines. By ninth grade, students will have a good understanding of measurement concepts and well-developed measurement skills. Electronic measurement instruments aid the student in collecting, storing, and analyzing real-time measurement data. High school students should be able to make reasonable estimates and sensible judgments about the precision and accuracy of value they obtain.

Students in high school will distinguish between precision and accuracy of measurements. With the widespread use of calculator and computer technologies for gathering and displaying data, students will understand that selections of scale and viewing window become important choices. Through logarithmic scaling, students will graphically represent some naturally occurring phenomena. Students will understand how unit analysis can be used to make decisions about which units are most appropriate.

RESEARCH BASE: Students who can use measuring instruments and procedures when asked to do so often do not use this ability while performing an investigation. Typically, a student asked to undertake an investigation and given a set of equipment that includes measuring instruments will make a qualitative comparison even though she might be competent to use the instruments in a different context (Black, 1990). It appears students often know how to take measurements, but not what or when.

Middle-school and even high-school students may have limited understanding about the nature and purpose of estimation. They often think it is inferior to exact computation and equate it with guessing (Sowder, 1992b), so that they do not believe estimation is useful (Sowder & Wheeler, 1989). Students who see estimation as a valuable tactic for obtaining information use estimation more frequently and successfully (Threadgill-Sowder, 1984).

Students of all ages often interpret graphs as literal pictures rather than as symbolic representations of the situations (Leinhardt, Zaslavsky, & Stein, 1990; McDermitt, Rosenquist, & Van Zee, 19987). Many students interpret distance/time graphs as the paths of actual journeys (Kerslake, 1981).

8-10 Measurement
ESSENTIAL FOCUS:

1. Students should know how to use algebraic formulas to determine an unknown measure. (8^{th}).
2. Students should know how to use unit (dimensional) analysis in measurement computations (9th).
3. Students should know how to distinguish between precision and accuracy in measurement situation (10^{th}).

RESEARCH BASE: GEOMETRY
GRADES 8 - 10

INTRODUCTION: High School students should develop capacity with several ways of representing geometric ideas. These presentations will allow multiple approaches to solve geometric problems. Geometry offers a means of describing, analyzing, and understanding the world; its ideas can be useful both in other areas of mathematics and in applied settings. By the ninth grade, students will have explored and discovered relationships among two-and three-dimensional geometric shapes. The student's high school experiences in geometry will enhance their ability to discover patterns and formulate conjectures. Technology will be a useful tool for accomplishing this task.

Students in high school will draw connections between geometry and algebra through the use of Cartesian and other coordinate systems. Transformations of two-dimensional figures will allow students to gain a better appreciation for geometric relationships. Students will extend their knowledge of all triangles to include applications of trigonometry. Students will represent problem situations using discrete structures such as finite graphs, matrices, vectors, flowcharts and networks. Through the use of technology, students will be able to model these concepts in many applications.

RESEARCH BASE: Probably the most comprehensive study of an alternative to traditional Euclidean geometry instruction is Usiskin's investigation of the feasibility of a transformational approach (Cosford & Usiskin, 1971; Usiskin, 1969). Neither approach was clearly superior overall. On some measures, particularly attitudinal, the transformational approach seemed more successful on some measures of achievement.

"It is not enough…to learn about properties of shapes and the vocabulary of geometry; they (students) must understand what geometry is and how it relates to the real world and other topics in mathematics. Research has shown that our students must be active learners engaged in the process of discovering conjecturing, and thinking at higher levels: (Fortunato, 1993).

8-10 Geometry
ESSENTIAL FOCUS:

1. Students should know how to apply geometric relationships using the coordinate system (9^{th}) .
2. Students should know how to apply trigonometric relationships among the angles and sides of a right triangle (9^{th}).
3. Students should know how to establish the validity of geometric conjectures using deduction (10^{th}).
4. Students should know how to analyze transformations in the rectangular coordinate system (10^{th}).

RESEARCH BASE: PATTERNS, FUNCTIONS & ALGEBRA STRAND
GRADES 8-10

INTRODUCTION: Algebra is the core of mathematics. High school students' experiences in mathematics should provide insights into algebraic abstractions and structures. These insights can help students develop a deeper understanding of real-world phenomena. By the ninth grade, students will have explored various ways of representing linear and non-linear relationships. In high school, students should develop an understanding of the algebraic properties that govern the manipulation of symbols in expressions, equations, and inequalities. At the same time, working in real-world contexts may help students make sense of the underlying mathematical concepts and may foster an appreciation of those concepts. Using technology, students can model and analyze a wide range of phenomena.

Students in high school will become competent with their use of algebra. They will create models that satisfy applications of exponential and other non-linear functions. The development of function notation will assist students to better understand the effects of translations on graphs. In addition, students will recognize the effects of parameter changes. Functions notation will also help students identify how a relation might be represented through the use of parametric equations. Having gained a deeper insight into the applications of mathematics, students ill are able to use technology to solve a variety of problems and to identify the reasonableness of the answers that they obtain.

RESEARCH BASE: Students have difficulty understanding how symbols are used in algebra (Kieran, 1992). They are often unaware of the arbitrariness of the letters chosen to represent variables in equations (Wagner, 1981). Middle-school and high-school students may regard the letters as shorthand for single objects, or as specific but unknown numbers, or as generalized numbers before they understand the as representations of variables (Kieran, 992).
Another study focused on the mathematical performance of upper secondary students who had regular and prolonged access to graphing calculators (Ruthven, 1990). These students developed specific calculator techniques for finding symbolic rules for graphically represented functions. Interestingly, the graphing-calculator group outperformed students who did not have such access on tasks that required symbolization.

A study involving beginning high school students in learning mathematical modeling while using a computer for symbolic manipulation also suggested conceptual gains without noticeable skill loss (Heid, Sheets, et al., 1988). In the Heid study, distinctive patterns of classroom interaction were noted in the experimental course. The activities that characterized the experimental course included: making, defending, and debating mathematical conjectures; interpreting and reasoning about mathematical representations; and suggesting and justifying mathematical models. (Heid, 1988).

8-10 Patterns, Functions and Algebra
ESSENTIAL FOCUS:

1. Students should know how to analyze relationships between linear equations and their graphs, connecting the meaning of intercepts and slope to the context of the situation (8^{th}).
2. Students should know to recognize and generate equivalent forms of algebraic expressions and equations with justifications (8^{th}).
3. Students should know how to apply graphing concepts and function notation to non-linear functions (9^{th}).
4. Students should know how logarithmic and exponential functions are related (10^{th}).

RESEARCH BASE: DATA ANALYSIS AND PROBABILITY
GRADES 8 - 10

INTRODUCTION: Upon entering high school, students should be familiar with designing simple surveys and experiments; gather data through the use of tables, charts and graphs; and summarizing that data in various ways. Students will have computed probabilities of simple and some compound events, and will have performed simulations, comparing the results of the simulations to predicted probabilities.

In grades 8 – 10, students should gain a deep understanding of the issues entailed in drawing conclusions in light of variability. They should learn to ask questions that will help them evaluate the quality of surveys, observational studies, and controlled experiments. Students can use their skills in algebra to model and analyze data, with increasing understanding of what it means to fit data well.

High school students should line probability to other topics in mathematics, especially counting techniques, area concepts, and relationships between functions and the area under their graphs. Students should learn to determine the probability of a sample statistic for a known population and draw simple inferences about a population from randomly generated samples.

RESEARCH BASE: Research has shown students in grades 8 - 10 expect their own judgment to be more reliable than information obtained from data (Hancock, Kaput and Goldsmith, 1992). In the later middle grades and high school, students should address the ideas of sample selection and statistical and statistical inference and begin to understand that there are ways of quantifying how certain one can be about statistical results.
Even researchers trained in the use of statistics entertain statistical misconceptions. For example, they may erroneously believe that when conducting a replication studys' (sic) even smaller sample sizes than the first study's are sufficient, since sample should be "representative" of the population regardless of their size (Tversky & Kahneman, 1971). If trained researches have trouble with statistical concepts, it should not surprise us that students have misconceptions of some of the most elementary concepts, such as mean and variance.

A basic problem appears to be understanding the distinction between a variable making no difference and a variable that is correlated with the outcome in the opposite way than the students initially conceived (Kuhn, et al., 1988).

8-10 Data Analysis and Probability
ESSENTIAL FOCUS:

1. Students should know how to find a line of best fit and use it to make appropriate predictions (8^{th}).
2. Students should know how to use fundamental counting principles to determine probabilities of independent and simple dependent events (9^{th}).
3. Students should know how to use technology to find an appropriate equation of best fit to model for the data (10^{th}).
4. Students should know how to compute probabilities of compound events (10^{th}).

Grade: Eight — Number, Number Sense and Operations Standard

Number, Number Sense Standard:
Students demonstrate number sense including an understanding of number systems and operations, and how they relate to one another. Students compute fluently and make reasonable estimates using paper and pencil, technology-supported and mental methods.

Mathematical Process Benchmarks Gr. 8 - 10:

8-10MPB A. Formulate a problem or mathematical model in response to a specific need or situation, determine information required to solve the problem, choose method for obtaining this information, and set limits for acceptable solution.

8-10MPB B. Apply mathematical knowledge and skills routinely in other content areas and practical situations.

8-10MPB C. Recognize and use connections between equivalent representations and related procedures for a mathematical concept; e.g., zero of a function and the x-intercept of the graph of the function, apply proportional thinking when measuring, describing functions, and comparing probabilities.

8-10MPB D. Apply reasoning processes and skills to construct logical verifications or counter-examples to test conjectures and to justify and defend algorithms and solutions.

8-10MPB E. Use a variety of mathematical representations flexibly and appropriately to organize, record and communicate mathematical ideas.

8-10MPB F. Use precise mathematical language and notations to represent problem situations and mathematical ideas.

8-10MPB G. Write clearly and coherently about mathematical thinking and ideas.

8-10MPB H. Locate and interpret mathematical information accurately, and communicate ideas, processes and solutions in a complete and easily understood manner.

Benchmarks Gr. 8 - 10:
Indicators Gr. 8:

8-10NB A. Use scientific notation to express large numbers and numbers less than one.
- 8NI 1. Use scientific notation to express large numbers and small numbers between 0 and 1.

8-10NB B. Identify subsets of the real number system.
- 8NI 2. Recognize that natural numbers, whole numbers, integers, rational numbers and irrational numbers are subsets of the real number system.

8-10NB C. Apply properties of operations and the real number system, and justify when they hold for a set of numbers.
- 8NI 4. Explain and use the inverse and identity properties and use inverse relationships (addition/subtraction, multiplication/division, squaring/square roots) in problem solving situations.

8-10NB D. Connect physical, verbal and symbolic representations of integers, rational numbers and irrational numbers.

8-10NB E. Compare, order and determine equivalent forms of real numbers.

8-10NB F. Explain the effect of operations on the magnitude of quantities.

8-10NB G. Estimate, compute and solve problems involving real numbers, including ratio, proportion and percent, and explain solutions.

8NI 5. Determine when an estimate is sufficient and when an exact answer is needed in problem situations, and evaluate estimates in relation to actual answers; e.g., very close, less than, greater than.

8NI 6. Estimate, compute and solve problems involving rational numbers, including ratio, proportion and percent, and judge the reasonableness of solutions.

8-10NB H. Find the square root of perfect squares, and approximate the square root of non-perfect squares.

8NI 7. Find the square root of perfect squares, and approximate the square root of non-perfect squares as consecutive integers between which the root lies; e.g., $\sqrt{130}$ is between 11 and 12.

8-10NB I. Estimate, compute and solve problems involving scientific notation, square roots and numbers with integer exponents.

8NI 3. Apply order of operations to simplify expressions and perform computations involving integer exponents and radicals.

8NI 8. Add, subtract, multiply, divide and compare numbers written in scientific notation.

Grade: Eight — Measurement Standard

Measurement Standard
Students estimate and measure to a required degree of accuracy and precision by selection and using appropriate units, tools and technologies.

Mathematical Process Benchmarks Gr. 8 - 10:

8-10MPB A. Formulate a problem or mathematical model in response to a specific need or situation, determine information required to solve the problem, choose method for obtaining this information, and set limits for acceptable solution.

8-10MPB B. Apply mathematical knowledge and skills routinely in other content areas and practical situations.

8-10MPB C. Recognize and use connections between equivalent representations and related procedures for a mathematical concept; e.g., zero of a function and the x-intercept of the graph of the function, apply proportional thinking when measuring, describing functions, and comparing probabilities.

8-10MPB D. Apply reasoning processes and skills to construct logical verifications or counter-examples to test conjectures and to justify and defend algorithms and solutions.

8-10MPB E. Use a variety of mathematical representations flexibly and appropriately to organize, record and communicate mathematical ideas.

8-10MPB F. Use precise mathematical language and notations to represent problem situations and mathematical ideas.

8-10MPB G. Write clearly and coherently about mathematical thinking and ideas.

8-10MPB H. Locate and interpret mathematical information accurately, and communicate ideas, processes and solutions in a complete and easily understood manner.

Benchmarks Gr. 8 -10:

Indicators Gr. 8:

8-10MB A. Solve increasingly complex non-routine measurement problems and check for reasonableness of results.

- 8MI 6. Solve and determine the reasonableness of the results for problems involving rates and derived measurements, such as velocity and density, using formulas, models and graphs.

8-10MB B. Use formulas to find surface area and volume for specified three-dimensional objects accurate to a specified level of precision.

- 8MI 3. Use appropriate levels of precision when calculating with measurements.
- 8MI 4. Derive formulas for surface area and volume and justify them using geometric models and common materials. For example, find:
  a. the surface area of a cylinder as a function of its height and radius;
  b. that the volume of a pyramid (or cone) is one-third of the volume of a prism (or cylinder) with the same base area and height.

8-10MB C. Apply indirect measurement techniques, tools and formulas, as appropriate, to find perimeter, circumference and area of circles, triangles, quadrilaterals and composite shapes, and to find volume of prisms, cylinders, and pyramids.

- 8MI 5. Determine surface area for pyramids by analyzing their parts.
- 8MI 9. Demonstrate understanding of the concepts of perimeter, circumference and area by using established formula for triangles, quadrilaterals, and circles to determine the surface area and volume of prisms, pyramids, cylinders, spheres and cones. (Note: Only volume should be calculated for spheres and cones.)

8-10MB	D.	Use proportional reasoning and apply indirect measurement techniques, including right triangle trigonometry and properties of similar triangles, to solve problems involving measurements and rates.	
		8MI 1.	Compare and order the relative size of common U.S. customary units and metric units; e.g., mile and kilometer, gallon and liter, pound and kilogram
		8MI 2.	Use proportional relationships and formulas to convert units from one measurement system to another; e.g., degrees Fahrenheit to degrees Celsius.
		8MI 7.	Apply proportional reasoning to solve problems involving indirect measurements or rates.
8-10MB	E.	Estimate and compute various attributes, including length, angle measure, area, surface area and volume, to a specified level of precision.	
		8MI 3.	Use appropriate levels of precision when calculating with measurements.
		8MI 8.	Find the sum of the interior and exterior angles of regular convex polygons with and without measuring the angles with a protractor.
		8MI 10.	Use conventional formulas to find the surface area and volume of prisms, pyramids and cylinders and the volume of spheres and cones to a specified level of precision.
8-10MB	F.	Write and solve real-world, multi-step problems involving money, elapsed time and temperature, and verify reasonableness of solutions.	
		8MI 6.	Solve and determine the reasonableness of the results for problems involving rates and derived measurements, such as velocity and density, using formulas, models and graphs.

Grade: Eight — Geometry and Spatial Sense Standard

Geometry and Spatial Sense Standard
Students identify, classify, compare and analyze characteristics, properties and relationships of one-, two- and three-dimensional geometric figures and objects. Students use spatial reasoning, properties of geometric objects, and transformations to analyze mathematical situations and solve problems.

Mathematical Process Benchmarks Gr. 8 - 10:

- 8-10MPB A. Formulate a problem or mathematical model in response to a specific need or situation, determine information required to solve the problem, choose method for obtaining this information, and set limits for acceptable solution.
- 8-10MPB B. Apply mathematical knowledge and skills routinely in other content areas and practical situations.
- 8-10MPB C. Recognize and use connections between equivalent representations and related procedures for a mathematical concept; e.g., zero of a function and the x-intercept of the graph of the function, apply proportional thinking when measuring, describing functions, and comparing probabilities.
- 8-10MPB D. Apply reasoning processes and skills to construct logical verifications or counter-examples to test conjectures and to justify and defend algorithms and solutions.
- 8-10MPB E. Use a variety of mathematical representations flexibly and appropriately to organize, record and communicate mathematical ideas.
- 8-10MPB F. Use precise mathematical language and notations to represent problem situations and mathematical ideas.
- 8-10MPB G. Write clearly and coherently about mathematical thinking and ideas.
- 8-10MPB H. Locate and interpret mathematical information accurately, and communicate ideas, processes and solutions in a complete and easily understood manner.

Benchmarks Gr. 8-10:
Indicators Gr. 8:

- 8-10GSB A. Formally define geometric figures.

- 8-10GSB B. Describe and apply the properties of similar and congruent figures; and justify conjectures involving similarity and congruence.
  - 8GSI 1. Make and test conjectures about characteristics and properties (e.g., sides, angles, symmetry) of two-dimensional figures and three-dimensional objects.
  - 8GSI 3. Use proportions in several forms to solve problems involving similar figures (part-to-part, part-to-whole, corresponding sides between figures).

- 8-10GSB C. Recognize and apply angle relationships in situations involving intersecting lines, perpendicular lines and parallel lines.
  - 8GSI 2. Recognize the angles formed and the relationship between the angles when two lines intersect and when parallel lines are cut by a transversal.

- 8-10GSB D. Use coordinate geometry to represent and examine the properties of geometric figures.
  - 8GSI 1. Make and test conjectures about characteristics and properties (e.g., sides, angles, symmetry) of two-dimensional figures and three-dimensional objects.
  - 8GSI 4. Represent and analyze shapes using coordinate geometry; e.g., given three vertices and the type of quadrilateral, find the coordinates of the fourth vertex.

8-10GSB	E.	Draw and construct representations of two-and three-dimensional geometric objects using a variety of tools, such as straightedge, compass and technology. 8GSI 6. Draw nets for a variety of prisms, pyramids, cylinders and cones.
8-10GSB	F.	Represent and model transformations in a coordinate plane and describe the results. 8GSI 5. Draw the results of translations, reflections, rotations and dilations of objects in the coordinate plane, and determine properties that remain fixed; e.g., lengths of sides remain the same under translations.
8-10GSB	G.	Prove or disprove conjectures and solve problems involving two- and three-dimensional objects represented within a coordinate system.
8-10GSB	H.	Establish the validity of conjectures about geometric objects, their properties and relationships by counter-example, inductive and deductive reasoning, and critiquing arguments made by others. 8GSI 1. Make and test conjectures about characteristics and properties (e.g., sides, angles, symmetry) of two-dimensional figures and three-dimensional objects.

Grade: Eight — Patterns, Functions and Algebra Standard

Patterns, Functions and Algebra Standard
Students use patterns, relations and functions to model, represent and analyze problem situations that involve variable quantities. Students analyze, model and solve problems using various representations such as tables, graphs and equations.

Mathematical Process Benchmarks Gr. 8 - 10:

- 8-10MPB A. Formulate a problem or mathematical model in response to a specific need or situation, determine information required to solve the problem, choose method for obtaining this information, and set limits for acceptable solution.
- 8-10MPB B. Apply mathematical knowledge and skills routinely in other content areas and practical situations.
- 8-10MPB C. Recognize and use connections between equivalent representations and related procedures for a mathematical concept; e.g., zero of a function and the x-intercept of the graph of the function, apply proportional thinking when measuring, describing functions, and comparing probabilities.
- 8-10MPB D. Apply reasoning processes and skills to construct logical verifications or counter-examples to test conjectures and to justify and defend algorithms and solutions.
- 8-10MPB E. Use a variety of mathematical representations flexibly and appropriately to organize, record and communicate mathematical ideas.
- 8-10MPB F. Use precise mathematical language and notations to represent problem situations and mathematical ideas.
- 8-10MPB G. Write clearly and coherently about mathematical thinking and ideas.
- 8-10MPB H. Locate and interpret mathematical information accurately, and communicate ideas, processes and solutions in a complete and easily understood manner.

Benchmarks Gr. 8-10:
Indicators Gr. 8:

- 8-10PFAB A. Generalize and explain patterns and sequences in order to find the next term and the *n*th term.
  - 8PFAI 2. Generalize patterns and sequences by describing how to find the *n*th term.
- 8-10PFAB B. Identify and classify functions as linear or nonlinear, and contrast their properties using tables, graphs or equations.
  - 8PFAI 3. Identify functions as linear or nonlinear based on information given in a table, graph or equation.
- 8-10PFAB C. Translate information from one representation (words, table, graph or equation) to another representation of a relation or function.
  - 8PFAI 1. Relate the various representations of a relationship; i.e., relate a table to graph, description and symbolic form.
- 8-10PFAB D. Use algebraic representations, such as tables, graphs, expressions, functions and inequalities, to model and solve problem situations.
  - 8PFAI 4. Extend the uses of variables to include covariants where *y* depends on *x*.
  - 8PFAI 5. Use physical models to add and subtract monomials and polynomials, and to multiply a polynomial by a monomial.
  - 8PFAI 8. Write, simplify and evaluate algebraic expressions (including formulas) to generalize situations and solve problems.
- 8-10PFAB E. Analyze and compare functions and their graphs using attributes, such as rates of change, intercepts and zeros.
  - 8PFAI 6. Describe the relationship between the graph of a line and its equation, including being able to explain the meaning of slope as a constant rate of change and *y*-intercept in real-world problems.

8-10PFAB F. Solve and graph linear equations and inequalities.

8PFAI 7. Use symbolic algebra (equations and inequalities), graphs and tables to represent situations and solve problems.

8PFAI 9. Solve linear equations and inequalities graphically, symbolically and using technology.

8-10PFAB G. Solve quadratic equations with real roots by graphing, formula and factoring.

8PFAI 12. Solve simple quadratic equations graphically; e.g., $y = x2 – 16$.

8-10PFAB H. Solve systems of linear equations involving two variables graphically and symbolically.

8PFAI 10. Solve 2 by 2 systems of linear equations graphically and by simple substitution.

8PFAI 11. Interpret the meaning of the solution of a 2 by 2 system of equations; i.e., point, line, no solution.

8-10PFAB I. Model and solve problem situations involving direct and inverse variation.

8PFAI 14. Differentiate and explain types of changes in mathematical relationships, such as linear vs. nonlinear, continuous vs. noncontinuous, direct variation vs. inverse variation.

8-10PFAB J. Describe and interpret rates of change from graphical and numerical data.

8PFAI 13. Compute and interpret slope, midpoint and distance given a set of ordered pairs.

8PFAI 15. Describe and compare how changes in an equation affects the related graphs; e.g., for a linear equation changing the coefficient of x affects the slope and changing the constant affects the intercepts.

8PFAI 16. Use graphing calculators or computers to analyze change; e.g., interest compounded over time as a nonlinear growth pattern.

Grade: Eight — Data Analysis and Probability Standard

Data Analysis and Probability Standard:
Students pose questions and collect, organize, represent, interpret and analyze data to answer those questions. Students develop and evaluate inferences, predictions and arguments that are based on data.

Mathematical Process Benchmarks Gr. 8 - 10:

- 8-10MPB A. Formulate a problem or mathematical model in response to a specific need or situation, determine information required to solve the problem, choose method for obtaining this information, and set limits for acceptable solution.
- 8-10MPB B. Apply mathematical knowledge and skills routinely in other content areas and practical situations.
- 8-10MPB C. Recognize and use connections between equivalent representations and related procedures for a mathematical concept; e.g., zero of a function and the x-intercept of the graph of the function, apply proportional thinking when measuring, describing functions, and comparing probabilities.
- 8-10MPB D. Apply reasoning processes and skills to construct logical verifications or counter-examples to test conjectures and to justify and defend algorithms and solutions.
- 8-10MPB E. Use a variety of mathematical representations flexibly and appropriately to organize, record and communicate mathematical ideas.
- 8-10MPB F. Use precise mathematical language and notations to represent problem situations and mathematical ideas.
- 8-10MPB G. Write clearly and coherently about mathematical thinking and ideas.
- 8-10MPB H. Locate and interpret mathematical information accurately, and communicate ideas, processes and solutions in a complete and easily understood manner.

Benchmarks Gr. 8-10:
Indicators Gr. 8:

- 8-10DPB A. Create, interpret and use graphical displays and statistical measures to describe data; e.g., box-and-whisker plots, histograms, scatterplots, measures of center and variability.
  - 8DPI 1. Use, create and interpret scatterplots and other types of graphs as appropriate.
- 8-10DPB B. Evaluate different graphical representations of the same data to determine which is the most appropriate representation for an identified purpose.
  - 8DPI 2. Evaluate different graphical representations of the same data to determine which is the most appropriate representation for an identified purpose; e.g., line graph for change over time, circle graph for part-to-whole comparison, scatterplot for relationship between two variants.
  - 8DPI 3. Differentiate between discrete and continuous data and appropriate ways to represent each.
- 8-10DPB C. Compare the characteristics of the mean, median and mode for a given set of data, and explain which measure of center best represents the data.
  - 8DPI 5. Explain the mean's sensitivity to extremes and its use in comparison with the median and mode.

8-10DPB	D.	Find, use and interpret measures of center and spread, such as mean and quartiles, and use those measures to compare and draw conclusions about sets of data.		
		8DPI	4.	Compare two sets of data using measures of center (mean, mode, median) and measures of spread (range, quartiles, interquartile range, percentiles).
8-10DPB	E.	Evaluate the validity of claims and predictions that are based on data by examining the appropriateness of the data collection and analysis.		
		8DPI	8.	Describe how the relative size of a sample compared to the target population affects the validity of predictions.
8-10DPB	F.	Construct convincing arguments based on analysis of data and interpretation of graphs.		
		8DPI	6.	Make conjectures about possible relationship in a scatterplot and approximate line of best fit.
		8DPI	9.	Construct convincing arguments based on analysis of data and interpretation of graphs.
		8DPI	7.	Identify different ways of selecting samples, such as survey response, random sample, representative sample and convenience sample.
		8DPI	10.	Calculate the number of possible outcomes for a situation, recognizing and accounting for when items may occur more than once or when order is important.
		8DPI	11.	Demonstrate an understanding that the probability of either of two disjoint events occurring can be found by adding the probabilities for each and that the probability of one independent event following another can be found by multiplying the probabilities.

Algebra — Number, Number Sense and Operations Standard

Number, Number Sense Standard:
Students demonstrate number sense including an understanding of number systems and operations, and how they relate to one another. Students compute fluently and make reasonable estimates using paper and pencil, technology-supported and mental methods.

The student will be able to meet any previous objective of this standard, and, in addition ….

Mathematical Processes Benchmarks Gr. 8 - 10:

AMPB A. Formulate a problem or mathematical model in response to a specific need or situation, determine information required to solve the problem, choose method for obtaining this information, and set limits for acceptable solution**.**
AMPB B. Apply mathematical knowledge and skills routinely in other content areas and practical situations.
AMPB C. Recognize and use connections between equivalent representations and related procedures for a mathematical concept; e.g., zero of a function and the x -intercept for the graph of the function and apply proportional thinking when measuring, describing functions, and comparing probabilities.
AMPB D. Apply reasoning processes and skills to justify and defend the validity of solution strategies and results.
AMPB E. Use a variety of mathematical representations flexibly and appropriately to organize, record and communicate mathematical ideas.
AMPB F. Use precise mathematical language and notations to represent problem situations and mathematical ideas.
AMPB G. Write clearly and coherently about mathematical thinking and ideas.
AMPB H. Locate and interpret mathematical information accurately, and communicate ideas, processes and solutions in a complete and easily understood manner.

Essential Focus:

1. Students should know how to apply graphing concepts and function notation to nonlinear functions.
2. Students should know how to analyze relationships between linear equations and their graphs connecting the meaning of intercepts and slope to the context of the situation.
3. Students should know how to recognize and generate equivalent forms of algebraic expressions and equations with justification.
4. Students should know how to apply graphing concepts and function notations to nonlinear functions.

Benchmark Gr. 8 - 10:
Indicators Gr. 9:

8-10ANB A. Use scientific notation to express large numbers and numbers less than one.

8-10ANB B. Identify subsets of the real number system

8-10ANB C. Apply properties of operations and the real number system and justify when they hold for a set of numbers.
9ANI 1. Identify and justify whether properties (closure, identity, inverse, commutative and associative) hold for a given set and operations; e.g., even integers and multiplication.

8-10ANB D. Connect physical, verbal and symbolic representations of integers, rational numbers and irrational numbers.
8-10ANB E. Compare, order and determine equivalent forms of real numbers.

	9ANI	3.	Compare, order and determine equivalent forms for rational and irrational numbers.
8-10ANB	F.		Explain the effects of operations on the magnitude of quantities.
	9ANI	3.	Explain the effects of operations such as multiplication or division, and of computing powers and roots on the magnitude of quantities.
8-10ANB	G.		Estimate, compute and solve problems involving real numbers, including ratio, proportion and percent, and explain solutions.
	9ANI	4.	Demonstrate fluency in computations using real numbers.
8-10ANB	H.		Find the square root of perfect squares, and approximate the square root of non-perfect squares.
8-10ANB	I.		Estimate, compute and solve problems involving scientific notation, square roots and numbers with integer exponents.
	9ANI	5.	Estimate the solutions for problem situations involving square and cube roots.

***Grade Level Indicators:**

Numbers and Number Systems

ANI	1.	Apply factorials to solve problems

Computation and Estimation

ANI	2.	Estimate sums, differences, products and quotients of real numbers.
ANI	3.	Estimate the nth root of a given number greater than zero between consecutive integers when n is an integer.
ANI	4.	Compare, order and determine equivalence of real numbers.
ANI	5.	Estimate answers, compute, and solve problems involving real numbers.
ANI	6.	Compare and contrast the real number system, the rational number system and the whole numbers system.
ANI	7.	Recognize, understand, spell, define and use vocabulary applicable to this standard.
ANI	8.	Use estimation to eliminate choices in multiple-choice tests.
ANI	9.	Use estimation to determine reasonableness of problem situations in a wide variety of applications.
ANI	10.	Estimate the shape of graphs of various functions and algebraic expressions.
ANI	11.	Use mental computation when computer and calculator are inappropriate.

* - Additional indicators from past course of study.

Measurement Standard
Students estimate and measure to a required degree of accuracy and precision by selection and using appropriate units, tools and technologies.

The student will be able to meet any previous objectives of this standard, and, in addition ….

Mathematical Processes Benchmarks Gr. 8 – 10:

AMPB A. Formulate a problem or mathematical model in response to a specific need or situation, determine information required to solve the problem, choose method for obtaining this information, and set limits for acceptable solution**.**

AMPB B. Apply mathematical knowledge and skills routinely in other content areas and practical situations.

AMPB C. Recognize and use connections between equivalent representations and related procedures for a mathematical concept; e.g., zero of a function and the x -intercept for the graph of the function and apply proportional thinking when measuring, describing functions, and comparing probabilities.

AMPB D. Apply reasoning processes and skills to justify and defend the validity of solution strategies and results.

AMPB E. Use a variety of mathematical representations flexibly and appropriately to organize, record and communicate mathematical ideas.

AMPB F. Use precise mathematical language and notations to represent problem situations and mathematical ideas.

AMPB G. Write clearly and coherently about mathematical thinking and ideas.

AMPB H. Locate and interpret mathematical information accurately, and communicate ideas, processes and solutions in a complete and easily understood manner.

Essential Focus:

1. Students should know how to apply graphing concepts and function notation to nonlinear functions.
2. Students should know how to analyze relationships between linear equations and their graphs connecting the meaning of intercepts and slope to the context of the situation.
3. Students should know how to recognize and generate equivalent forms of algebraic expressions and equations with justification.
4. Students should know how to apply graphing concepts and function notations to nonlinear functions.

Benchmarks Gr. 8 – 10:
Indicators Gr. 9:

8-10AMB A. Solve increasingly complex non-routine measurement problems and check for reasonableness of results.

8-10AMB B. Use formulas to find surface area and volume for specified three-dimensional objects accurate to a specified level of precision.

8-10AMB C. Apply indirect measurement techniques, tools and formulas, as appropriate, to find perimeter, circumference and area of circles, triangles, quadrilaterals and composite shapes, and the find volume of prisms, cylinders, and pyramids.

8-10AMB D. Use proportional reasoning and apply indirect measurement techniques, including right triangle trigonometry and properties of similar

triangles, to solve problems involving measurements and rates.

9AMI 1. Convert rates within the same measurement system; e.g., miles per hour to feet per second; kilometers per hour to meters per second.

9AMI 2. Use unit analysis to check computations involving measurement.

9AMI 4. Use scale drawings and right triangle trigonometry to solve problems that include unknown distances and angle measures.

9AMI 5. Solve problems involving unit conversion for situations involving distances, areas, volumes and rates within the same measurement system.

8-10AMB E. Estimate and compute areas and volume in increasingly complex problem situations.

8-10AMB F. Write and solve real world, multi-step problems involving money, elapsed time and temperature, and verify reasonableness of solutions.

***Grade Level Indicators:**

Measurement Units

AMI 1. Estimate and use measurements.

AMI 2. Understand the need for measurement and the probability that any measurement is accurate to some designated specification.

AMI 3. Establish ratios with and without common units.

AMI 4. Understand and solve right triangle relationships as they relate to measurement----specifically those that deal with the Pythagorean Theorem.

AMI 5. Graph and interpret ordered pairs.

AMI 6. Compare total sales from a variety of items.

AMI 7. Develop an ability to identify real problems and provide possible solutions to these problems.

AMI 8. Determine area and volume.

AMI 9. Recognize, understand, spell, define and use vocabulary applicable to this strand.

Use Measurement Techniques and Tools

AMI 10. Use the ratio of lengths in similar two-dimensional figures or three-dimensional objects to calculate the ratio of their areas or volumes respectively.

* - Additional indicators from past course of study.

Geometry and Spatial Sense Standard
Students identify, classify, compare and analyze characteristics, properties and relationships of one-, two- and three-dimensional geometric figures and objects. Students use spatial reasoning, properties of geometric objects, and transformations to analyze mathematical situations and solve problems.

Mathematical Processes Benchmarks Gr. 8 – 10 :

AMPB A. Formulate a problem or mathematical model in response to a specific need or situation, determine information required to solve the problem, choose method for obtaining this information, and set limits for acceptable solution**.**
AMPB B. Apply mathematical knowledge and skills routinely in other content areas and practical situations.
AMPB C. Recognize and use connections between equivalent representations and related procedures for a mathematical concept; e.g., zero of a function and the x -intercept for the graph of the function and apply proportional thinking when measuring, describing functions, and comparing probabilities.
AMPB D. Apply reasoning processes and skills to justify and defend the validity of solution strategies and results.
AMPB E. Use a variety of mathematical representations flexibly and appropriately to organize, record and communicate mathematical ideas.
AMPB F. Use precise mathematical language and notations to represent problem situations and mathematical ideas.
AMPB G. Write clearly and coherently about mathematical thinking and ideas.
AMPB H. Locate and interpret mathematical information accurately, and communicate ideas, processes and solutions in a complete and easily understood manner.

Essential Focus:

1. Students should know how to apply graphing concepts and function notation to nonlinear functions.
2. Students should know how to analyze relationships between linear equations and their graphs connecting the meaning of intercepts and slope to the context of the situation.
3. Students should know how to recognize and generate equivalent forms of algebraic expressions and equations with justification.
4. Students should know how to apply graphing concepts and function notations to nonlinear functions.

Benchmarks Gr. 8 - 10:
Indicators Gr. 9:

8-10AGSB A. Formally define geometric figures.

8-10AGSB B. Describe and apply the properties of similar and congruent figures; and justify conjectures involving similarity and congruence.

8-10AGSB C. Recognize and apply angle relationships in situations involving intersecting lines, perpendicular lines and parallel lines.

8-10AGSB D. Use coordinate geometry to represent and examine the properties of geometric figures.

8-10AGSB E. Draw and construct representations of two- and three-dimensional geometric objects using a variety of tools, such as straightedge, compass and technology.

8-10AGSB F. Represent and model transformations in a coordinate plane and describe the results.

8-10AGSB	G.	Prove or disprove conjectures and solve problems involving two and three-dimensional objects represented within a coordinate system.
	9AGSI 3.	Analyze two-dimensional figures in a coordinate plane; e.g., use slope and distance formulas to show that a quadrilateral is a parallelogram.
8-10AGSB	H.	Establish the validity of conjectures about geometric objects, their properties and relationships by counter-example, inductive and deductive reasoning, and critiquing arguments made by others.
8-10AGSB	I.	Use right triangle trigonometric relationships to determine lengths and angle measures.
	9AGSI 1.	Define the basic trigonometric ratios in right triangles: sine, cosine and tangent.

Patterns, Functions and Algebra Standard
Students use patterns, relations and functions to model, represent and analyze problem situations that involve variable quantities. Students analyze, model and solve problems using various representations such as tables, graphs and equations**.**

The student will be able to meet any previous objective of this standard, and, in addition ….

Mathematical Processes Benchmarks Gr. 8 - 10:

AMPB A. Formulate a problem or mathematical model in response to a specific need or situation, determine information required to solve the problem, choose method for obtaining this information, and set limits for acceptable solution**.**
AMPB B. Apply mathematical knowledge and skills routinely in other content areas and practical situations.
AMPB C. Recognize and use connections between equivalent representations and related procedures for a mathematical concept; e.g., zero of a function and the x -intercept for the graph of the function and apply proportional thinking when measuring, describing functions, and comparing probabilities.
AMPB D. Apply reasoning processes and skills to justify and defend the validity of solution strategies and results.
AMPB E. Use a variety of mathematical representations flexibly and appropriately to organize, record and communicate mathematical ideas.
AMPB F. Use precise mathematical language and notations to represent problem situations and mathematical ideas.
AMPB G. Write clearly and coherently about mathematical thinking and ideas.
AMPB H. Locate and interpret mathematical information accurately, and communicate ideas, processes and solutions in a complete and easily understood manner.

Essential Focus:

1. Students should know how to apply graphing concepts and function notation to nonlinear functions.
2. Students should know how to analyze relationships between linear equations and their graphs connecting the meaning of intercepts and slope to the context of the situation.
3. Students should know how to recognize and generate equivalent forms of algebraic expressions and equations with justification.
4. Students should know how to apply graphing concepts and function notations to nonlinear functions**.**

Benchmarks Gr. 8 - 10:
Indicators Gr. 9 & 10:

8-10APFAB A. Generalize and explain patterns and sequences in order to find the next term and the *n*th term.
9APFAI 2. Generalize patterns using functions or relationships (linear, quadratic and exponential), and freely translate among tabular, graphical and symbolic representations.

8-10APFAB B. Identify and classify functions as linear or nonlinear, and contrast their properties using tables, graphs or equations.
9APFAI 1. Define function with ordered pairs in which each domain element is assigned exactly one range element.
9APFAI 3. Describe problem situations (linear, quadratic and exponential) by using tabular, graphical and symbolic representations.

8-10APFAB	C.	Translate information from one representation (words, table, graph or equation) to another representation of a relation or function.
		9APFAI 2. Generalize patterns using functions or relationships (linear, quadratic and exponential), and freely translate among tabular, graphical and symbolic representations.
8-10APFAB	D.	Use algebraic representations, such as tables, graphs, expressions, functions and inequalities, to model and solve problem situations.
		9APFAI 7. Use formulas to solve problems involving exponential growth and decay.
		9APFAI 11. Add, subtract, multiply and divide monomials and polynomials (division of polynomials by monomials only).
		9APFAI 12. Simplify rational expressions by eliminating common factors and applying properties of integer exponents.
8-10APFAB	E.	Analyze and compare functions and their graphs using attributes, such as rates of change, intercepts and zeros.
		9APFAI 4. Demonstrate the relationship among zeros of a function, roots of equations, and solutions of equations graphically and in words.
		9APFAI 5. Describe and compare characteristics of the following families of functions: linear, quadratic and exponential functions; e.g., general shape, number of roots, domain, range, rate of change, maximum or minimum.
8-10APFAB	F.	Solve and graph linear equations and inequalities.
		9APFAI 6. Write and use equivalent forms of equations and inequalities in problem situations; e.g., changing a linear equation to the slope-intercept form.
		9APFAI 8. Find linear equations that represent lines that pass through a given set of ordered pairs, and find linear equations that represent lines parallel or perpendicular to a given line through a specific point.
		10APFAI 10. Solve real-world problems that can be modeled using linear, quadratic, and exponential or square root functions.
8-10APFAB	G.	Solve quadratic equations with real roots by graphing, formula and factoring.
		9APFAI 10. Solve quadratic equations with real roots by factoring, graphing, using the quadratic formula and with technology.
		10APFAI 8. Graph the quadratic relationship that defines circles.
		10APFAI 10. Solve real-world problems that can be modeled using linear, quadratic, and exponential or square root functions.
8-10APFAB	H.	Solve systems of linear equations involving two variables graphically and symbolically.
		9APFAI 9. Solve and interpret the meaning of 2 by 2 systems of linear equations graphically, by substitution and by elimination, with and without technology.
		10APFAI 7. Solve systems of linear inequalities.
		10APFAI 11. Solve real-world problems that can be modeled, using systems of linear equations and inequalities.
8-10APFAB	I.	Model and solve problem situations involving direct and inverse variation.
		9APFAI 13. Model and solve problems involving direct and inverse variation using proportional reasoning.
		9APFAI 14. Describe the relationship between slope and the graph of a direct variation and inverse variation.
8-10APFAB	J.	Describe and interpret rates of change from graphical and numerical data.

9APFAI 15. Describe how a change in the value of a constant in a linear or quadratic equation affects the related graphs.
10APFAI 9. Recognize and explain that the slopes of parallel lines are equal and the slopes of perpendicular lines are negative reciprocals.

***Grade Level Indicators:**

Use Patterns, Relations And Functions

APFAI 1. Define function formally and with f(x) notation.

Use Algebraic Representations

APFAI 2. Solve equations and inequalities having rational expressions as coefficients and solutions

Analyze Change

APFAI 3. Model real-world phenomena with polynomial and exponential functions.
APFAI 4. Explore the relationship between zeros and intercepts of functions.
APFAI 5. Translate among tables, algebraic expressions and graphs of functions.
APFAI 6. Use a graphing calculator or computer to generate the graph of a function.
APFAI 7. Explore the relation between a linear function and its inverse.
APFAI 8. Describe the general characteristics of polynomial functions and use them in problem-solving situations.
APFAI 9. Explore the conic sections and their graphs using a graphing calculator or computer.
APFAI 10. Apply trigonometric functions to problem situations involving triangles.
APFAI 11. Discover general relationships between the algebraic description of a conic and special properties of this conic.
APFAI 12. Recognize, understand, spell, define, and use vocabulary applicable to this standard.
APFAI 13. Describe problem situations by using and relating numerical, symbolic, and graphical representations.
APFAI 14. Use the language and notation of functions in symbolic and graphic settings.
APFAI 15. Recognized use the equivalent ideas of zeros of a function, roots of an equation, and solutions of an equation in terms of graphical and symbolic representations.
APFAI 16. Describe and use the logic of equivalence in working with equations, inequalities, and functions.
APFAI 17. Develop graphical techniques of solution for problem situations involving functions.
APFAI 18. Explore and describe characterizing features of functions.
APFAI 19. Make arguments and proofs in algebraic settings.
APFAI 20. Factor the difference between two squares.
APFAI 21. Determine slope, midpoint, and distance.
APFAI 22. Explore and combine rational functions.
APFAI 23. Explore factoring techniques.
APFAI 24. Solve quadratic equations by factoring and formula.
APFAI 25. Set up and solve linear equations.
APFAI 26. Solve systems of linear equations with two variables.
APFAI 27. Describe geometric situations and phenomena using variables, equations, and functions.
APFAI 28. Describe measures of central tendency, mean median, mode, and variance, algebraically and graphically.
APFAI 29. Represent inequalities on the number line and in the coordinate lane.
APFAI 30. Use the coordinate arguments in making geometric proofs.
APFAI 31. Symbolize transformations of figures and graphs.
APFAI 32. Explore the geometric basis for the functions of trigonometry.

APFAI 33. Graph linear functions.
APFAI 34. Recognize, understand, spell, define and use vocabulary applicable to this standard.

* - Additional indicators from past course of study.

Data Analysis and Probability Standard:
Students pose questions and collect, organize, represent, interpret and analyze data to answer those questions. Students develop and evaluate inferences, predictions and arguments that are based on data.

The student will be able to meet any previous objective of this standard and, in addition ….

Mathematical Processes Benchmarks Gr. 8 - 10:

AMPB A. Formulate a problem or mathematical model in response to a specific need or situation, determine information required to solve the problem, choose method for obtaining this information, and set limits for acceptable solution**.**
AMPB B. Apply mathematical knowledge and skills routinely in other content areas and practical situations.
AMPB C. Recognize and use connections between equivalent representations and related procedures for a mathematical concept; e.g., zero of a function and the x -intercept for the graph of the function and apply proportional thinking when measuring, describing functions, and comparing probabilities.
AMPB D. Apply reasoning processes and skills to justify and defend the validity of solution strategies and results.
AMPB E. Use a variety of mathematical representations flexibly and appropriately to organize, record and communicate mathematical ideas.
AMPB F. Use precise mathematical language and notations to represent problem situations and mathematical ideas.
AMPB G. Write clearly and coherently about mathematical thinking and ideas.
AMPB H. Locate and interpret mathematical information accurately, and communicate ideas, processes and solutions in a complete and easily understood manner.

Essential Focus:

1. Students should know how to apply graphing concepts and function notation to nonlinear functions.
2. Students should know how to analyze relationships between linear equations and their graphs connecting the meaning of intercepts and slope to the context of the situation.
3. Students should know how to recognize and generate equivalent forms of algebraic expressions and equations with justification.
4. Students should know how to apply graphing concepts and function notations to nonlinear functions.

Benchmarks Gr. 8 - 10:
Indicators Gr. 9 & 10:

8-10ADPB A. Create, interpret and use graphical displays and statistical measures to describe data; e.g., box-and-whisker plots, histograms, scatterplots, measures of center and variability.
9ADPI 1. Classify data as univariate (single variable) or bivariate (two variables) and as quantitative (measurement) or qualitative (categorical) data.
9ADPI 2. Create a scatterplot for a set of bivariate data, sketch the line of best fit, and interpret the slope of the line of best fit.
9ADPI 3. Analyze and interpret frequency distributions based on spread, symmetry, skewness, clusters and outliers.
10ADPI 2. Represent and analyze bivariate data using appropriate graphical displays (scatterplots, parallel box-and-whisker plots, histograms with more than one set of data, tables, charts, spreadsheets) with and without technology.
10ADPI 3. Display bivariate data where at least one variable is categorical.
10ADPI 4. Identify outliers on a data display; e.g., use interquartile range to identify outliers on a box-and-whisker plot.

10ADPI 6. Interpret the relationship between two variables using multiple graphical displays and statistical measures; e.g., scatterplots, parallel box-and-whisker plots, and measures of center and spread.

8-10ADPB B. Evaluate different graphical representations of the same data to determine which is the most appropriate representation for an identified purpose.

8-10ADPB C. Compare the characteristics of the mean, median and mode for a given set of data, and explain which measure of center best represents the data.

10ADPI 1. Describe measures of center and the range verbally, graphically and algebraically.

8-10ADPB D. Find, use and interpret measures of center and spread, such as mean and quartiles, and use those measures to compare and draw conclusions about sets of data.

10ADPI 6. Interpret the relationship between two variables using multiple graphical displays and statistical measures; e.g., scatterplots, parallel box-and-whisker plots, and measures of center and spread.

8-10ADPB E. Evaluate the validity of claims and predictions that are based on data by examining the appropriateness of the data collection and analysis.

9ADPI 4. Describe and compare various types of studies (survey, observation, experiment), and identify possible misuses of statistical data.

8-10ADPB F. Construct convincing arguments based on analysis of data and interpretation of graphs.

9ADPI 6. Make inferences about relationships in bivariant data, and recognize the difference between evidence of relationship (correlation) and causation.

8-10ADPB G. Describe sampling methods and analyze the effects of method chosen on how well the resulting sample represents the population.

9ADPI 5. Describe characteristics and limitations of sampling methods, and analyze the effects of random versus biased sampling; e.g., determine and justify whether the sample is likely to be representative of the population.

10ADPI 5. Provide examples and explain how a statistic may or may not be an attribute of the entire population; e.g., intentional or unintentional bias may be present.

8-10ADPB H. Use counting techniques, such as permutations and combinations, to determine the total number of options and possible outcomes.

9ADPI 7. Use counting techniques and the Fundamental Counting principle to determine the total number of possible outcomes for mathematical situations.

8-10ADPB I. Design an experiment to test a theoretical probability, and record and explain results.

9ADPI 8. Describe, create and analyze a sample space and use it to calculate probability.

8-10ADPB J. Compute probabilities of compound events, independent events, and simple dependent events.

8-10ADPB K. Make predictions based on theoretical probabilities and experimental results.

9ADPI 10. Use theoretical and experimental probability, including simulations or random numbers, to estimate probabilities and to solve problems dealing with uncertainty; e.g., compound events, independent events, simple dependent events.

***Grade Level Indicators:**

Data Collection

ADPI 1. Identify situations involving independent and dependent events, and explain differences between and common misconceptions about probabilities associated with those events.

* - Additional indicators from past course of study.

Honors Algebra

The standards, benchmarks, and indicators for Honors Algebra are the same as those of Algebra outlined earlier in this document. With honors students the following expectations are included:

1. Go at a faster pace
2. Cover the entire text
3. Use explorations, extensions, and all related computer and graphing calculator materials, including some programming on the TI-83 and TI-92.
4. Localize applications.
5. Supplement content with extended topics that will vary according to background and interests of both the instructor and the students.

Geometry — Number, Number Sense and Operations Standard

Number, Number Sense Standard:
Students demonstrate number sense including an understanding of number systems and operations, and how they relate to one another. Students compute fluently and make reasonable estimates using paper and pencil, technology-supported and mental methods.

The student will be able to meet any previous grade level indicators and, in addition….

Mathematical Processes Benchmarks Gr. 8 - 10 :

GMPB A. Formulate a problem or mathematical model in response to a specific need or situation, determine information required to solve the problem, choose method for obtaining this information, and set limits for acceptable solution**.**

GMPB B. Apply mathematical knowledge and skills routinely in other content areas and practical situations.

GMPB C. Recognize and use connections between equivalent representations and related procedures for a mathematical concept; e.g., zero of a function and the x -intercept for the graph of the function and apply proportional thinking when measuring, describing functions, and comparing probabilities.

GMPB D. Apply reasoning processes and skills to justify and defend the validity of solution strategies and results.

GMPB E. Use a variety of mathematical representations flexibly and appropriately to organize, record and communicate mathematical ideas.

GMPB F. Use precise mathematical language and notations to represent problem situations and mathematical ideas.

GMPB G. Write clearly and coherently about mathematical thinking and ideas.

GMPB H. Locate and interpret mathematical information accurately, and communicate ideas, processes and solutions in a complete and easily understood manner.

Essential Focus:

1. Students should know how to apply geometric relationships using the coordinate system.
2. Students should know how to apply trigonometric relationships between the angles and sides of a right triangle.
3. Students shoulder know how to establish the validity of geometric conjectures using deduction and critique arguments made by others.
4. Students should know how to analyze transformations in the rectangular coordinates.

Benchmarks Gr. 8 – 10:
Indicators 8-10:

8-10GNB A. Use scientific notation to express large numbers and numbers less than one.

8-10GNB B. Identify subsets of the real number system.

8-10GNB C. Apply properties of operations and the real number system and justify when they hold for a set of numbers.

8-10GNB D. Connect physical, verbal and symbolic representations of integers, rational numbers and irrational numbers.

10GNI 1. Connect physical, verbal and symbolic representations of irrational numbers; e.g., construct $\sqrt{2}$ as a hypotenuse or on a number line.

8-10GNB E. Compare, order and determine equivalent forms of real numbers.
9GNI 2. Compare, order and determine equivalence of real numbers.(*), and rational and irrational numbers.

8-10GNB F. Explain the effects of operations on the magnitude of quantities.

8-10GNB G. Estimate, compute and solve problems involving real numbers, including ratio, proportion and percent, and explain solutions.
9GNI 4. Demonstrate fluency of computation with real numbers; i.e. positive and negative rational numbers, positive and negative irrational numbers.

8-10GNB H. Find the square root of perfect squares, and approximate the square root of non-perfect squares.

8-10GNB I. Estimate, compute and solve problems involving scientific notation, square roots and numbers with integer exponents.

***Grade Level Indicators:**

Meaning of Operations

GNI 1. Explain the meaning of the *n*th root.

Computation and Estimation

GNI 2. Use factorial notation and computations to represent and solve problem situations involving arrangements.
GNI 3. Approximate the *n*th root of a given number greater than zero between consecutive integers when *n* is an integer; e.g., the 4^{th} root of 50 is between 2 and 3.
GNI 4. Solve problems and make applications involving percents.
GNI 5. Solve and use proportions.
GNI 6. Understand, represent, and use numbers written in a variety of equivalence forms in real-world and mathematical problem-solving situations.
GNI 7 Develop an understanding of operations with integers using the number line and other models of integers.
GNI 8. Solve problems and make applications involving percentages.
GNI 9. Estimate answers, compute, and solve problems involving real numbers.
GNI 10. Use an open sentence (algebraic equation) to symbolize a problem situation and solve the equation to find a solution to the problem.
GNI 11. Validate and generalize problem situations.
GNI 12. Select appropriate notation and methods for symbolizing the problem statement and the solution process.

* - Additional indicators from past course of study.

Geometry — Measurements Standard

Measurement Standard
Students estimate and measure to a required degree of accuracy and precision by selection and using appropriate units, tools and technologies.

The student will be able to meet any previous objective of this standard, and, in addition….

Mathematical Processes Benchmarks Gr. 8 - 10:

GMPB A. Formulate a problem or mathematical model in response to a specific need or situation, determine information required to solve the problem, choose method for obtaining this information, and set limits for acceptable solution**.**
GMPB B. Apply mathematical knowledge and skills routinely in other content areas and practical situations.
GMPB C. Recognize and use connections between equivalent representations and related procedures for a mathematical concept; e.g., zero of a function and the x -intercept for the graph of the function and apply proportional thinking when measuring, describing functions, and comparing probabilities.
GMPB Use inductive and deductive reasoning to draw conclusions and make predictions.
GMPB D. Apply reasoning processes and skills to justify and defend the validity of solution strategies and results.
GMPB E. Use a variety of mathematical representations flexibly and appropriately to organize, record and communicate mathematical ideas.
GMPB F. Use precise mathematical language and notations to represent problem situations and mathematical ideas.
GMPB G. Write clearly and coherently about mathematical thinking and ideas.
GMPB H. Locate and interpret mathematical information accurately, and communicate ideas, processes and solutions in a complete and easily understood manner.

Essential Focus:

1. Students should know how to apply geometric relationships using the coordinate system.
2. Students should know how to apply trigonometric relationships between the angles and sides of a right triangle.
3. Students shoulder know how to establish the validity of geometric conjectures using deduction and critique arguments made by others.
4. Students should know how to analyze transformations in the rectangular coordinates.

Benchmarks Gr. 8 - 10:
Indicators Gr. 8 – 10:

8-10GMB A. Solve increasingly complex non-routine measurement problems and check for reasonableness of results.
- 9GMI 1. Determine the number of significant digits in a measurement.
- 10GMI 1. Explain how a small error in measurement may lead to a large error in calculated results.
- 10GMI 2. Calculate relative error.
- 10GMI 3. Explain the difference between absolute error and relative error in measurement.

8-10GMB B. Use formulas to find surface are and volume for specified three-dimensional objects accurate to a specified level pf precision.

8-10GMB C. Apply direct measurement techniques, tools and formulas, as appropriate, to find perimeter, circumference and area of circles, triangles, quadrilaterals and composite shapes, and to find volume of prisms, cylinders, and pyramids.

9GMI 1. Derive a formula for the surface area of a cone as a function of its slant height and the circumference of its base.

8-10GMB D. Use proportional reasoning and apply indirect measurement techniques including right triangle trigonometry and properties of similar triangles, to solve problems involving measurements and rates.

9GMI 2. Use scale drawings and right triangle trigonometry to solve problems that include unknown distances and angle measures.

9GMI 3. Use the ratio of lengths in similar two-dimensional figures or three-dimensional objects to calculate the ratio of their areas or volumes respectively.

9GMI 4. Solve problems involving unit conversion for situations involving distances, areas, volumes and rates within the same measurement system.

10GMI 5. Determine the measures of central and inscribed angles and their associated major and minor arcs.

8-10GMB E. Estimate and compute various attributes, including length, angle measure area, surface area and volume, to a specified level of precision.

8-10GMB F. Write and solve real-world, multi-step problems involving money, elapsed time and temperature, and verify reasonableness of solutions.

***Grade Level Indicators:**

Use Measurement Techniques & Tools

GMI 1. Give examples of how the same absolute error can be problematic in one situation but not in another; e.g., compare "accurate to the nearest foot" when measuring the height of a person versus when measuring the height of a mountain.

Measurement Units

GMI 2. Determine the number of significant digits in a measurement.

GMI 3. State and apply area formulas for the following regions: circular, rectangular, parallelogram, trapezoidal and triangular.

GMI 4. Apply volume formulas for the following: prisms, cylinders, and spheres.

GMI 5. Select and compute with appropriate standard or metric units to measure length, area, volume, weight, capacity, time, money and temperature.

GMI 6. Make appropriate judgments regarding accuracy and precision.

GMI 7. Determine formulas for surface area.

GMI 8. Measure and compute perimeters for irregular polygonal figures.

GMI 9. Compute area for regular polygonal regions, other composite figures, and lattice (geoboard) figures.

GMI 10. Make appropriate measurements and compute volumes of solids such as prisms, cylinders, pyramids and cones.

GMI 11. Successfully utilize a ruler and protractor in measurement tasks.

GMI 12. Read a scale on a measurement device to the nearest mark and make interpolations where appropriate.

GMI 13. Estimate and use measurements

GMI 14. Understand the need for measurement and the probability that any measurement is accurate to some designated specification.

GMI 15. Establish ratios with and without common units.

GMI 16. Understand and solve right triangle relationships as they relate to measurement---specifically those that deal with the Pythagorean theorem.

GMI 17. Graph and interpret ordered pairs.

GMI 18. Develop an ability to identify real problems and provide possible solutions to these problems.

*Additional indicators from past course of study.

Geometry and Spatial Sense Standard
Students identify, classify, compare and analyze characteristics, properties and relationships of one-, two- and three-dimensional geometric figures and objects. Students use spatial reasoning, properties of geometric objects, and transformations to analyze mathematical situations and solve problems

The student will be able to meet any previous objective of this standard, and, in addition….

Mathematical Processes Benchmarks Gr. 8 - 10:

GMPB A. Formulate a problem or mathematical model in response to a specific need or situation, determine information required to solve the problem, choose method for obtaining this information, and set limits for acceptable solution**.**
GMPB B. Apply mathematical knowledge and skills routinely in other content areas and practical situations.
GMPB C. Recognize and use connections between equivalent representations and related procedures for a mathematical concept; e.g., zero of a function and the x -intercept for the graph of the function and apply proportional thinking when measuring, describing functions, and comparing probabilities.
GMPB Use inductive and deductive reasoning to draw conclusions and make predictions.
GMPB D. Apply reasoning processes and skills to justify and defend the validity of solution strategies and results.
GMPB E. Use a variety of mathematical representations flexibly and appropriately to organize, record and communicate mathematical ideas.
GMPB F. Use precise mathematical language and notations to represent problem situations and mathematical ideas.
GMPB G. Write clearly and coherently about mathematical thinking and ideas.
GMPB H. Locate and interpret mathematical information accurately, and communicate ideas, processes and solutions in a complete and easily understood manner.

Essential Focus:

1. Students should know how to apply geometric relationships using the coordinate system.
2. Students should know how to apply trigonometric relationships between the angles and sides of a right triangle.
3. Students should know how to develop geometric relationships using the coordinate system.
4. Students should know how to apply trigonometric relationships among the angles and sides of a right triangle.
5. Students should know how to establish the validity of geometric conjectures using deduction.
6. Students should know how to analyze transformations in the rectangular coordinate system.

Benchmarks Gr . 8 - 10:
Indicators Gr. 8 – 10:

8-10GGSB A. Formally define geometric figures.
10GGSI 2. Formally define and explain key aspects of geometric figures, including:
a. interior and exterior angles of polygons;
b. segments related to triangles (median, altitude, midsegment);
c. points of concurrency related to triangles (centroid, incenter, orthocenter, circumcenter);
d. circles (radius, diameter, chord, circumference, major arc, minor arc, sector, segment, inscribed angle).
10GGSI 3. Recognize and explain the necessity for certain terms to remain undefined, such as point, line and plane.

10GGSI 10. Solve problems involving chords, radii and arcs within the same circle.

8-10GGSB B. Describe and apply the properties of similar and congruent figures; and justify conjectures involving similarity and congruence.

8-10GGSB C. Recognize and apply angle relationships in situations involving intersecting lines, perpendicular lines and parallel lines.
10GGSI 10. Solve problems involving chords, radii and arcs within the same circle.

8-10GGSB D. Use coordinate geometry to represent and examine the properties of geometric figures.

8-10GGSB E. Draw and construct representations of two- and three-dimensional geometric objects using a variety of tools, such as straightedge, compass and technology.
10GGSI 4. Construct right triangles, equilateral triangles, parallelograms, trapezoids, rectangles, rhombuses, squares and kites, using compass and straightedge or dynamic geometry software.
10GGSI 5. Construct congruent figures and similar figures using tools, such as compass, straightedge, and protractor or dynamic geometry software.

8-10GGSB F. Represent and model transformations in a coordinate plane and describe the results.
10GGSI 8. Derive coordinate rules for translations, reflections and rotations of geometric figures in the coordinate plane.
10GGSI 9. Show and describe the results of combinations of translations, reflections and rotations (compositions); e.g., perform compositions and specify the result of a composition as the outcome of a single motion, when applicable.

8-10GGSB G. Prove or disprove conjectures and solve problems involving two and three-dimensional objects represented within a coordinate system.
9GGSI 3. Analyze two-dimensional figures in a coordinate plane; e.g., use slope and distance formulas to show that a quadrilateral is a parallelogram.

8-10GGSB H. Establish the validity of conjectures about geometric objects, their properties and relationships by counter-example, inductive and deductive reasoning, and critiquing arguments made by others.
10GGSI 3. Make, test and establish the validity of conjectures about geometric properties and relationships using counterexample, inductive and deductive reasoning, and paragraph or two-column proof, including:
a. prove the Pythagorean Theorem;
b. prove theorems involving triangle similarity and congruence;
c. prove theorems involving properties of lines, angles, triangles and quadrilaterals;
d. test a conjecture using basic constructions made with a compass and straightedge or technology.
10GGSI 10. Solve problems involving chords, radii and arcs within the same circle.

8-10GGSB I. Use right triangle trigonometric relationships to determine lengths and angle measures.
9GGSI 1. Define the basic trigonometric ratios in right triangles: sine, cosine and tangent.
9GGSI 2. Apply proportions and right triangle trigonometric ratios to solve problems involving missing lengths and angle measures in similar figures.

***Grade Level Indicators:**

Transformation & Symmetry

GGSI 1. Perform reflections and rotations using compass and straightedge constructions and dynamic geometry software.
GGSI 2. Describe multiplication of a vector and a scalar graphically and algebraically, and apply to problem situations.
GGSI 3. Explore and verbalize relationships between different kinds of geometric figures.
GGSI 4. Develop a minimum set of properties that describe a geometric figure.
GGSI 5. Build a model of a figure given top, side, and front views.
GGSI 6. Validate fundamental geometric theorems using manipulative materials and informal arguments.
GGSI 7. Calculate missing measurements of similar figures.
GGSI 8. Graph similar figures, reflections, translations, and rotations on a coordinate plane.
GGSI 9. Extend experiences validating fundamental geometric theorems.
GGSI 10. Find perimeters (circumferences) and areas of polygons (circles).
GGSI 11. Explore the uses of the Pythagorean theorem.
GGSI 12. Find the surface areas and volumes of basic geometric solids.
GGSI 13. Create and interpret drawings of three-dimensional objects.
GGSI 14. Represent problem situations with geometric models and apply properties of figures.
GGSI 15. Describe and apply properties of similar and congruent figures.
GGSI 16. Demonstrate an understanding of angles and parallel and perpendicular lines.
GGSI 17. Explore inductive and deductive reasoning through applications to various subject areas.
GGSI 18. Translate between synthetic and coordinate representations.
GGSI 19. Identify congruent and similar figures using transformations with computer programs.
GGSI 20. Deduce properties of figures using transformations and using coordinates.
GGSI 21. Use deductive reasoning.
GGSI 22. Explore compass and straightedge constructions in the context of geometric theorems.
GGSI 23. Demonstrate an ability to use proof.
GGSI 24. Use a variety of techniques of proof, such as synthetic, transformational, and coordinate.
GGSI 25. Use a variety of proof formats, including T- (or two-column) proof and the paragraph proof.
GGSI 26. Explore different strategies of proof.
GGSI 27 Investigate different proofs of theorems.
GGSI 28. Develop an understanding of an axiomatic system.
GGSI 29. Apply transformations and coordinates in problem solving.
GGSI 30. Represent problem situations with geometric models and apply properties of figures.

GMI	31.	Express and apply different types of measurement scales.
GMI	32.	Determine area and volume.
GMI	33.	Use an open sentence (algebraic equation) to symbolize a problem situation and solve the equation to find a solution to the problem).
GMI	34.	Validate and generalize problem situations.
GMI	35.	Select appropriate notation and methods for symbolizing the problem statement and the solution process.

*Additional indicators from past course of study.

Patterns, Functions and Algebra Standard
Students use patterns, relations and functions to model, represent and analyze problem situations that involve variable quantities. Students analyze, model and solve problems using various representations such as tables, graphs and equations.

Mathematical Processes Benchmarks Gr. 8 - 10:

GMPB A. Formulate a problem or mathematical model in response to a specific need or situation, determine information required to solve the problem, choose method for obtaining this information, and set limits for acceptable solution**.**

GMPB B. Apply mathematical knowledge and skills routinely in other content areas and practical situations.

GMPB C. Recognize and use connections between equivalent representations and related procedures for a mathematical concept; e.g., zero of a function and the x -intercept for the graph of the function and apply proportional thinking when measuring, describing functions, and comparing probabilities.

GMPB D. Apply reasoning processes and skills to justify and defend the validity of solution strategies and results.

GMPB E. Use a variety of mathematical representations flexibly and appropriately to organize, record and communicate mathematical ideas.

GMPB F. Use precise mathematical language and notations to represent problem situations and mathematical ideas.

GMPB G. Write clearly and coherently about mathematical thinking and ideas.

GMPB H. Locate and interpret mathematical information accurately, and communicate ideas, processes and solutions in a complete and easily understood manner.

Essential Focus:

1. Students should know how to apply geometric relationships using the coordinate system.
2. Students should know how to apply trigonometric relationships between the angles and sides of a right triangle.
3. Students should know how to establish the validity of geometric conjectures using deduction and critique arguments made by others.
4. Students should know how to analyze transformations in the rectangular coordinates.

Benchmarks Gr. 8 - 10:
Indicators Gr. 8 – 10:

8-10GPFAB A. Generalize and explain patterns and sequences in order to find the next term and the *n*th term.

8-10GPFAB B. Identify and classify functions as linear or nonlinear, and contrast their properties using tables, graphs or equations.
9GPFAI 3. Describe problem situations by using and relating numerical, symbolic, and graphical representations.

8-10GPFAB C. Translate information from one representation (words, table, graph or equation) to another representation of a relation or function.

8-10GPFAB D. Use algebraic representations, such as tables, graphs, expressions, functions and inequalities, to model and solve problem situations.

8-10GPFAB E. Analyze and compare functions and their graphs using attributes, such as rates of change, intercepts and zeros.
9GPFAI 3. Explore the relationship between zeros and intercepts of functions.

8-10GPFAB F. Solve and graph linear equations and inequalities.

8-10GPFAB G. Solve quadratic equations with real roots by graphing, formula and factoring.
9GPFAI 10. Solve quadratic equations by factoring and formula.
10GPFAI 8. Graph the quadratic relationship that defines circles.

8-10GPFAB H. Solve systems of linear equations involving two variables graphically and symbolically.

8-10GPFAB I. Model and solve problem situations involving direct and inverse variation.

8-10GPFAB J. Describe and interpret rates of change from graphical and numerical data.

***Grade Level Indicators:**

Use Algebraic Representations

GPFAI 1. Model real-world phenomena with polynomial and exponential functions.
GPFAI 2. Translate among tables, algebraic expressions and graphs of functions.
GPFAI 3. Use a graphing calculator or computer to generate the graph of a function.
GPFAI 4. Explore the relation between a linear function and its inverse.
GPFAI 5. Describe the general characteristics of polynomial functions and use them in problem-solving situations.
GPFAI 6. Explore the conic sections and their graphs using a graphing calculator or computer.
GPFAI 7. Apply trigonometric functions to problem situations involving triangles.
GPFAI 8. Discover general relationships between the algebraic description of a conic and special properties of this conic.
GPFAI 9. Recognize, understand, spell, define, and use vocabulary applicable to this strand.
GPAFI 10. Use the language and notation of functions in symbolic and graphic settings.
GPFAI 11. Recognize, relate, and use the equivalent ideas of zeros of a function, roots of an equation, and solutions of an equation in terms of graphical and symbolic representations.
GPFAI 12. Describe and use the logic of equivalence in working with equations, inequalities, and functions.
GPFAI 13. Develop graphical techniques of solution for problem situations involving functions.
GPFAI 14. Explore and describe characterizing features of functions.
GPFAI 15. Make arguments and proofs in algebraic settings.
GPFAI 16. Factor the difference between two squares.
GPFAI 17. Determine slope, midpoint, and distance.
GPFAI 18. Explore and combine rational functions.
GPFAI 19. Explore factoring techniques.
GPFAI 20. Set up and solve linear equations.
GPFAI 21. Solve systems of linear equations with two variables.
GPFAI 22. Describe geometric situations and phenomena using variables, equations, and functions.
GPFAI 23. Describe measures of central tendency, mean median, mode, and variance, algebraically and graphically.
GPFAI 24. Represent inequalities on the number on the number line and in the coordinate lane.

GPFAI	25.	Use the coordinate arguments in making geometric proofs.
GPFAI	26.	Symbolize transformations of figures and graphs.
GPFAI	27.	Explore the geometric basis for the functions of trigonometry.
GPFAI	28.	Graph linear functions.
GPFAI	29.	Use an open sentence (algebraic equation) to symbolize a problem situation and solve the equation to find a solution to the problem.
GPFAI	30.	Validate and generalize problem situations.

*Additional indicators from past course of study.

Data Analysis and Probability Standard:
Students pose questions and collect, organize, represent, interpret and analyze data to answer those questions. Students develop and evaluate inferences, predictions and arguments that are based on data.

The student will be able to meet any previous objective of this standard, and, in addition….

Mathematical Processes Benchmarks Gr. 8 – 10:

GMPB A. Formulate a problem or mathematical model in response to a specific need or situation, determine information required to solve the problem, choose method for obtaining this information, and set limits for acceptable solution**.**

GMPB B. Apply mathematical knowledge and skills routinely in other content areas and practical situations.

GMPB C. Recognize and use connections between equivalent representations and related procedures for a mathematical concept; e.g., zero of a function and the x -intercept for the graph of the function and apply proportional thinking when measuring, describing functions, and comparing probabilities.

GMPB D. Apply reasoning processes and skills to justify and defend the validity of solution strategies and results.

GMPB E. Use a variety of mathematical representations flexibly and appropriately to organize, record and communicate mathematical ideas.

GMPB F. Use precise mathematical language and notations to represent problem situations and mathematical ideas.

GMPB G. Write clearly and coherently about mathematical thinking and ideas.

GMPB H. Locate and interpret mathematical information accurately, and communicate ideas, processes and solutions in a complete and easily understood manner.

Essential Focus:

1. Students should know how to use the fundamental counting principle to determine the probabilities of independent and simple dependent events.
2. Students should know how to use technology to find an appropriate equation of best fit to model the data.
3. Students should know how to compute probabilities of compound events.
4. Students should know how to find a line of best fit and use it to make appropriate predictions**.**

Benchmarks Gr. 8 – 10:
Indicators Gr. 8 – 10:

8-10GDPB A. Create, interpret and use graphical displays and statistical measures to describe data; e.g., box-and-whisker plots, histograms, scatterplots, measures of center and variability.

8-10GDPB B. Evaluate different graphical representations of the same data to determine which is the most appropriate representation for an identified purpose.

8-10GDPB C. Compare the characteristics of the mean, median and mode for a given set of data, and explain which measure of center best represents the data.

8-10GDPB	D.	Find, use and interpret measures of center and spread, such as mean and quartiles, and use those measures to compare and draw conclusions about sets of data.
8-10GDPB	E.	Evaluate the validity of claims and predictions that are based on data by examining the appropriateness of the data collection and analysis.
8-10GDPB	F.	Construct convincing arguments based on analysis of data and interpretation of graphs.
8-10GDPB	G.	Describe sampling methods and analyze the effects of method chosen on how well the resulting sample represents the population.
8-10GDPB	H.	Use counting techniques, such as permutations and combination, to determine the total number of options and possible outcomes.
8-10GDPB	I.	Design an experiment to test a theoretical probability, and record and explain results.
8-10GDPB	J.	Compute probabilities of compound events, independent events, and simple dependent events.
8-10GDPB	K.	Make predictions based on theoretical probabilities and experimental results.

***Grade Level Indicators:**

Probability

GDPI	1.	Model problems dealing with uncertainty with area models (geometric probability).

*Additional Indicator from past course of study.

Honors Geometry

The standards, benchmarks, and indicators for Honors Geometry are the same as those of Geometry outlined earlier in this document. With the honors students, the following expectations are included:

1. Go at a faster pace
2. Cover the entire text
3. Use explorations, extensions, and all related computer and graphing calculator materials.
4. Localize applications.
5. Supplement content with extended topics that will vary according to background and interests of both the instructor and the students.
6. Participate in selected math team competitions such as the Ohio Math League, OCTM Math Contest, AMC 10 National Math Contest.

RESEARCH BASE: NUMBERS AND OPERATIONS
GRADES 11-12

INTRODUCTION: Students in grades 11 - 12 should see number and operations from a more global perspective. Their understanding of number is the foundation for their understanding of algebra, the core of all mathematics. High school students should understand more fully the concept of a number system, how different number systems are related, and whether the properties of one system hold in another. Students will develop an increased ability to estimate the results of an arithmetic computation and judge the reasonableness of results obtained through technology.

Students in high school will understand the meaning of exponents and how to apply their properties in computations. The real number system will be explored for its use in matrices and vectors. High school students will relate complex numbers to problems for which there are no real solutions. Students will apply these concepts in a variety of problem-solving situations.

RESEARCH BASE: Regardless of the particular method used, students should be able to explain their method, understand that many methods exist, and see the usefulness of methods that are efficient, accurate and general. Students also need to be able to estimate and judge the reasonableness of results. Computational fluency should develop in tandem with understanding of the role of meaning of arithmetic operations in number systems (Hiebert, et al., 1997; Thornton, 1990).

11-12 Numbers and Number Sense
ESSENTIAL FOCUS:

1. Students should know how to use rational exponents (11^{th}).

RESEARCH BASE: MEASUREMENT
GRADES 11 - 12

INTRODUCTION: Opportunities to use and understand measurement arise naturally during high school in various disciplines. By ninth grade, students will have a good understanding of measurement concepts and well-developed measurement skills. Electronic measurement instruments aid the student in collecting, storing, and analyzing real-time measurement data. High school students should be able to make reasonable estimates and sensible judgments about the precision and accuracy of value they obtain.

Students in high school will distinguish between precision and accuracy of measurements. With the widespread use of calculator and computer technologies for gathering and displaying data, students will understand that selections of scale and viewing window become important choices. Through logarithmic scaling, students will graphically represent some naturally occurring phenomena. Students will understand how unit analysis can be used to make decisions about which units are most appropriate.

RESEARCH BASE: Students who can use measuring instruments and procedures when asked to do so often do not use this ability while performing an investigation. Typically, a student asked to undertake an investigation and given a set of equipment that includes measuring instruments will make a qualitative comparison even though she might be competent to use the instruments in a different context (Black, 1990). It appears students often know how to take measurements, but not what or when.

Middle-school and even high-school students may have limited understanding about the nature and purpose of estimation. They often think it is inferior to exact computation and equate it with guessing (Sowder, 1992b), so that they do not believe estimation is useful (Sowder & Wheeler, 1989). Students who see estimation as a valuable tactic for obtaining information use estimation more frequently and successfully (Threadgill-Sowder, 1984).

Students of all ages often interpret graphs as literal pictures rather than as symbolic representations of the situations (Leinhardt, Zaslavsky, & Stein, 1990; McDermitt, Rosenquist, & Van Zee, 19987). Many students interpret distance/time graphs as the paths of actual journeys (Kerslake, 1981).

11-12 Measurement
ESSENTIAL FOCUS:

1. Students should know how to analyze error approximation. (11^{th}).
2. Students should know how to use successive approximations to understand the concept of limit (12^{th}).

RESEARCH BASE: GEOMETRY
GRADES 11 - 12

INTRODUCTION: High School students should develop capacity with several ways of representing geometric ideas. These presentations will allow multiple approaches to solve geometric problems. Geometry offers a means of describing, analyzing, and understanding the world; its ideas can be useful both in other areas of mathematics and in applied settings. The student's high school experiences in geometry will enhance their ability to discover patterns and formulate conjectures. Technology will be a useful tool for accomplishing this task.

Students in high school will draw connections between geometry and algebra through the use of Cartesian and other coordinate systems. Transformations of two-dimensional figures will allow students to gain a better appreciation for geometric relationships. Students will extend their knowledge of all triangles to include applications of trigonometry. Students will represent problem situations using discrete structures such as finite graphs, matrices, vectors, flowcharts and networks. Through the use of technology, students will be able to model these concepts in many applications.

RESEARCH BASE: Probably the most comprehensive study of an alternative to traditional Euclidean geometry instruction is Usiskin's investigation of the feasibility of a transformational approach (Cosford & Usiskin, 1971; Usiskin, 1969). Neither approach was clearly superior overall. On some measures, particularly attitudinal, the transformational approach seemed more successful on some measures of achievement.
"It is not enough…to learn about properties of shapes and the vocabulary of geometry; they (students) must understand what geometry is and how it relates to the real world and other topics in mathematics. Research has shown that our students must be active learners engaged in the process of discovering conjecturing, and thinking at higher levels: (Fortunato, 1993).

11-12 Geometry
ESSENTIAL FOCUS:

1. Students should know how to use trigonometric relationships to determine lengths and angle measures in any triangle. (11^{th}).
2. Students should know how to visualize and represent objects in three-space (12^{th}).

RESEARCH BASE: PATTERNS, FUNCTIONS & ALGEBRA STRAND
GRADES 11 - 12

INTRODUCTION: Algebra is the core of mathematics. High school students' experiences in mathematics should provide insights into algebraic abstractions and structures. These insights can help students develop a deeper understanding of real-world phenomena. By the ninth grade, students will have explored various ways of representing linear and non-linear relationships. In high school, students should develop an understanding of the algebraic properties that govern the manipulation of symbols in expressions, equations, and inequalities. At the same time, working in real-world contexts may help students make sense of the underlying mathematical concepts and may foster an appreciation of those concepts. Using technology, students can model and analyze a wide range of phenomena.

Students in high school will become competent with their use of algebra. They will create models that satisfy applications of exponential and other non-linear functions. The development of function notation will assist students to better understand the effects of translations on graphs. In addition, students will recognize the effects of parameter changes. Functions notation will also help students identify how a relation might be represented through the use of parametric equations. Having gained a deeper insight into the applications of mathematics, students are able to use technology to solve a variety of problems and to identify the reasonableness of the answers that they obtain

.

RESEARCH BASE: Students have difficulty understanding how symbols are used in algebra (Kieran, 1992). They are often unaware of the arbitrariness of the letters chosen to represent variables in equations (Wagner, 1981). Middle-school and high-school students may regard the letters as shorthand for single objects, or as specific but unknown numbers, or as generalized numbers before they understand the as representations of variables (Kieran, 992).
Another study focused on the mathematical performance of upper secondary students who had regular and prolonged access to graphing calculators (Ruthven, 1990). These students developed specific calculator techniques for finding symbolic rules for graphically represented functions. Interestingly, the graphing-calculator group outperformed students who did not have such access on tasks that required symbolization.

A study involving beginning high school students in learning mathematical modeling while using a computer for symbolic manipulation also suggested conceptual gains without noticeable skill loss (Heid, Sheets, et al., 1988). In the Heid study, distinctive patterns of classroom interaction were noted in the experimental course. The activities that characterized the experimental course included: making, defending, and debating mathematical conjectures; interpreting and reasoning about mathematical representations; and suggesting and justifying mathematical models. (Heid, 1988).

11-12 Patterns, Functions and Algebra
ESSENTIAL FOCUS:

1. Students should know the properties and characteristics of polynomial and trigonometric functions (11^{th}).
2. Students should know how to create multiple symbolic representations of a function or relation that satisfied given conditions (12^{th}).

RESEARCH BASE: DATA ANALYSIS AND PROBABILITY
GRADES 11 - 12

INTRODUCTION: Upon entering high school, students should be familiar with designing simple surveys and experiments; gather data through the use of tables, charts and graphs; and summarizing that data in various ways. Students will have computed probabilities of simple and some compound events, and will have performed simulations, comparing the results of the simulations to predicted probabilities.

In grades 11 - 12, students should gain a deep understanding of the issues entailed in drawing conclusions in light of variability. They should learn to ask questions that will help them evaluate the quality of surveys, observational studies, and controlled experiments. Students can use their skills in algebra to model and analyze data, with increasing understanding of what it means to fit data well.

High school students should line probability to other topics in mathematics, especially counting techniques, area concepts, and relationships between functions and the area under their graphs. Students should learn to determine the probability of a sample statistic for a known population and draw simple inferences about a population from randomly generated samples.

RESEARCH BASE: Research has shown students in grades 11 - 12 expect their own judgment to be more reliable than information obtained from data (Hancock, Kaput and Goldsmith, 1992). In the later middle grades and high school, students should address the ideas of sample selection and statistical and statistical inference and begin to understand that there are ways of quantifying how certain one can be about statistical results.
Even researchers trained in the use of statistics entertain statistical misconceptions. For example, they may erroneously believe that when conducting a replication studys' (sic) even smaller sample sizes than the first study's are sufficient, since sample should be "representative" of the population regardless of their size (Tversky & Kahneman, 1971). If trained researches have trouble with statistical concepts, it should not surprise us that students have misconceptions of some of the most elementary concepts, such as mean and variance.

A basic problem appears to be understanding the distinction between a variable making no difference and a variable that is correlated with the outcome in the opposite way than the students initially conceived (Kuhn, et al., 1988).

11-12 Data Analysis and Probability
ESSENTIAL FOCUS:

1. Students should know how to find functions that model data (11^{th}).
2. Students should know how to create, implement and interpret simulations to model a statistical study (12^{th}).

Algebra II — Number, Number Sense and Operations Standard

Number, Number Sense Standard:
Students demonstrate number sense including an understanding of number systems and operations, and how they relate to one another. Students compute fluently and make reasonable estimates using paper and pencil, technology-supported and mental methods.

The students will be able to meet any previous objective of this standard, and, in addition ….

Mathematical Processes Benchmarks Gr. 11-12:

AIIMPB	A.	Construct algorithms for multi-step and non-routine problems.
AIIMPB	B.	Construct logical verifications or counter-examples to test conjectures and to justify or refute algorithms and solutions to problems.
AIIMPB	C.	Assess the adequacy and reliability of information available to solve a problem.
AIIMPB	D.	Select and use various types of reasoning and methods of proof.
AIIMPB	E.	Evaluate a mathematical argument and use reasoning and logic to judge its validity.
AIIMPB	F.	Present complete and convincing arguments and justifications, using inductive and deductive reasoning, adapted to be effective for various audiences.
AIIMPB	G.	Understand the difference between a statement that is verified by mathematical proof, such as a theorem, and one that is verified empirically using examples or data.
AIIMPB	H.	Use formal mathematical language and notation to represent ideas, to demonstrate relationships within and among representation systems, and to formulate generalizations.
AIIMPB	I.	Communicate mathematical ideas orally and in writing with a clear purpose and appropriate for a specific audience.
AIIMPB	J.	Apply mathematical modeling to workplace and consumer situations, including problem formulation, identification of a mathematical model, interpretation of solution within the model, and validation to original problem situation.

Essential Focus:

1. Students should know the properties and characteristics of polynomial and trigonometric functions.
2. Students should know how to create multiple symbolic representations of a function or relation that satisfies given conditions.

Benchmarks Gr. 11-12:
Indicators Gr. 11:

11-12AIINB	A.	Demonstrate that vectors and matrices are systems having some of the same properties of the real number system
	11AIINI 2.	Identify and explain the properties of vector addition and multiplication, and scalar multiplication, and justify your conclusions.
11-12AIINB	B.	Develop an understanding of properties of an representations for addition and multiplication of vectors and matrices.
	11AIINI 5.	Model (using the ordinate plane) vector addition and scalar multiplication.
11-12AIINB	C.	Apply factorials and exponents, including fractional exponents, to solve practical problems.

11AIINI 8. Apply factorials and numbers with negative or fractional exponents to solve problems.

11-12AIINB D. Demonstrate fluency in operations with real numbers, vectors and matrices, using mental computation or paper and pencil calculations for simple cases and technology for more complicated cases.
11AIINI 4. Use matrices to represent given information in a problem and solve that problem with technology.
11AIINI 6. Compute sums, differences, and products of matrices, using paper-and-pencil calculations for simple cases and technology for more complicated cases.

11-12AIINB E. Represent and compute with complex numbers.
11AIINI 3. Represent complex numbers on the complex plane.
11AIINI 7. Compute sums, differences, products and quotients of complex numbers.

***Grade Level Indicators:**

Number and Number Systems
AIINI 1. Explain the properties of matrix addition and multiplication and justify your conclusions.

Meaning of Operations
AIINI 2. Explain the meaning of the nth root.

Computation & Estimation
AIINI 3. Use vector addition and scalar multiplication to solve problems.
AIINI 4. Apply factorials to solve problems
AIINI 5. Estimate the *n*th root of a given number greater than zero between consecutive integers when n is an integer.
AIINI 6. Compare, order and determine equivalence of real numbers.
AIINI 7. Estimate answers, compute and solve problems involving real numbers.
AIINI 8. Compare and contrast the real number system, the rational number system and the whole number system.
AIINI 9. Extend his understanding to the complex number system and develop facility with its operations.

* - Additional indicators from past course of study.

Measurement Standard
Students estimate and measure to a required degree of accuracy and precision by selection and using appropriate units, tools and technologies.

The student will be able to meet any previous objective of this standard, and, in addition….

Mathematical Processes Benchmarks Gr. 11-12:

AIIMPB	A.	Construct algorithms for multi-step and non-routine problems.
AIIMPB	B.	Construct logical verifications or counter-examples to test conjectures and to justify or refute algorithms and solutions to problems.
AIIMPB	C.	Assess the adequacy and reliability of information available to solve a problem.
AIIMPB	D.	Select and use various types of reasoning and methods of proof.
AIIMPB	E.	Evaluate a mathematical argument and use reasoning and logic to judge its validity.
AIIMPB	F.	Present complete and convincing arguments and justifications, using inductive and deductive reasoning, adapted to be effective for various audiences.
AIIMPB	G.	Understand the difference between a statement that is verified by mathematical proof, such as a theorem, and one that is verified empirically using examples or data.
AIIMPB	H.	Use formal mathematical language and notation to represent ideas, to demonstrate relationships within and among representation systems, and to formulate generalizations.
AIIMPB	I.	Communicate mathematical ideas orally and in writing with a clear purpose and appropriate for a specific audience.
AIIMPB	J.	Apply mathematical modeling to workplace and consumer situations, including problem formulation, identification of a mathematical model, interpretation of solution within the model, and validation to original problem situation.

Essential Focus:

1. Students should know how to analyze error approximation.
2. Students should know how to use successive approximations to understand the concept of limit.

Benchmarks Gr. 8 - 10:
Indicators Gr. 10:

8-10AIIMB	A.	Solve increasingly complex non-routine measurement problems and check for reasonableness of results.		
		10AIIMI	2.	Calculate relative error.
		10AIIMI	3	Explain the difference between absolute error and relative error in measurement.
		10AIIMI	4.	Give examples of how the same absolute error can be problematic in one situation but not in another; e.g., compare "accurate to the nearest foot" when measuring the height of a person versus when measuring the height of a mountain.
8-10AIIMB	B.	Use formulas to find surface area and volume for specified three-dimensional objects accurate to a specified level of precision.		
8-10AIIMB	C.	Apply indirect measurement techniques, tools and formulas, as appropriate, to find perimeter, circumference and area of circles, triangles, quadrilaterals and composite shapes, and the find volume of prisms, cylinders, and pyramids.		

8-10AIIMB D. Use proportional reasoning and apply indirect measurement techniques, including right triangle trigonometry and properties of similar triangles, to solve problems involving measurements and rates.

8-10AIIMB E. Estimate and compute areas and volume in increasingly complex problem situations.

8-10AIIMB F. Write and solve real world, multi-step problems involving money, elapsed time and temperature, and verify reasonableness of solutions.

Benchmarks Gr. 11 – 12:
Indicators Gr. 11:

11-12AIIMB A. Explain differences among accuracy, precision and error, and describe how each of those can affect solutions in measurement situations.

11-12AIIMB B. Apply various measurement scales to describe e phenomena and solve problems.
11AIIMI 2. Use radian and degree angle measures to solve problems and perform conversions as needed.

11-12AIIMB C. Estimate and compute areas and volume in increasingly complex problem situations.
11AIIMI 3. Derive a formula for the surface area of a cone as a function of its slant height and the circumference of its base.
11AIIMI 4. Calculate distances, areas, surface areas and volumes of composite three-dimensional objects to a specified number of significant digits.

11-12AIIMB D. Solve problem situations involving derived measurements; e.g., density, acceleration.

***Grade Level Indicators:**

Measurement Units

AIIMI 1. Determine the number of significant digits in a measurement.

Use Measurement Techniques & Tools

AIIMI 2. Estimate and use measurements.
AIIMI 3. Understand the need for measurement and the probability that any measurement is accurate to some designated specification.
AIIMI 4. Understand and apply measurement concepts of distance-rate-time problems and acceleration problems only with real -world experiments.
AIIMI 5. Establish ratios with and without common units.
AIIMI 6. Understand and solve right triangle relationships as they relate to measurement --- specifically those that deal with the Pythagorean Theorem.
AIIMI 7. Graph and interpret ordered pairs.
AIIMI 8. Develop an ability to identify real problems and provide possible solutions to these problems.
AIIMI 9. Express and apply different types of measurement scales.

*Additional indicators from past course of study.

Geometry and Spatial Sense Standard
Students identify, classify, compare and analyze characteristics, properties and relationships of one-, two- and three-dimensional geometric figures and objects. Students use spatial reasoning, properties of geometric objects, and transformations to analyze mathematical situations and solve problems.

The students will be able to meet any previous objective of this standard, and, in addition ….

Mathematical Processes Benchmarks Gr. 11-12:

AIIMPB	A.	Construct algorithms for multi-step and non-routine problems.
AIIMPB	B.	Construct logical verifications or counter-examples to test conjectures and to justify or refute algorithms and solutions to problems.
AIIMPB	C.	Assess the adequacy and reliability of information available to solve a problem.
AIIMPB	D.	Select and use various types of reasoning and methods of proof.
AIIMPB	E.	Evaluate a mathematical argument and use reasoning and logic to judge its validity.
AIIMPB	F.	Present complete and convincing arguments and justifications, using inductive and deductive reasoning, adapted to be effective for various audiences.
AIIMPB	G.	Understand the difference between a statement that is verified by mathematical proof, such as a theorem, and one that is verified empirically using examples or data.
AIIMPB	H.	Use formal mathematical language and notation to represent ideas, to demonstrate relationships within and among representation systems, and to formulate generalizations.
AIIMPB	I.	Communicate mathematical ideas orally and in writing with a clear purpose and appropriate for a specific audience.
AIIMPB	J.	Apply mathematical modeling to workplace and consumer situations, including problem formulation, identification of a mathematical model, interpretation of solution within the model, and validation to original problem situation.

Essential Focus:

1. Students should know how to use trigonometric relationships to determine lengths and angle measures in any triange.
2. Students should know how to visualize and represent objects in 3-space.

Benchmarks Gr. 8 - 10 :
Indicators Gr. 9 & 10:

8-10AIIGSB	A.	Formally define geometric figures.		
		10AIIGSI	2.	Identify, sketch and classify the cross sections of three-dimensional objects.
8-10AIIGSB	B.	Describe and apply the properties of similar and congruent figures; and justify conjectures involving similarity and congruence.		
8-10AIIGSB	C.	Recognize and apply angle relationships in situations involving intersecting lines, perpendicular lines and parallel lines.		

9AIIGSI 2. Apply proportions and right triangle trigonometric ratios to solve problems involving missing lengths and angle measures in similar figures.

8-10AIIGSB D. Use coordinate geometry to represent and examine the properties of geometric figures.
10AIIGSI 2. Identify, sketch and classify the cross sections of three-dimensional objects.

8-10AIIGSB E. Draw and construct representations of two- and three-dimensional geometric objects using a variety of tools, such as straightedge, compass and technology.
10AIIGSI 2. Identify, sketch and classify the cross sections of three-dimensional objects.

8-10AIIGSB F. Represent and model transformations in a coordinate plane and describe the results.
10AIIGSI 6. Identify the reflection and rotation symmetries of two- and three-dimensional figures.
10AIIGSI 8. Derive coordinate rules for translations, reflections and rotations of geometric figures in the coordinate plane.

8-10AIIGSB G. Prove or disprove conjectures and solve problems involving two and three-dimensional objects represented within a coordinate system.

8-10AIIGSB H. Establish the validity of conjectures about geometric objects, their properties and relationships by counter-example, inductive and deductive reasoning, and critiquing arguments made by others.

8-10AIIGSB I. Use right triangle trigonometric relationships to determine lengths and angle measures.

Benchmarks Gr. 11-12:
Indicators Gr. 11:

11-12AIIGSB A. Use trigonometric relationships to verity and determine solutions in problem situations.
11AIIGSI 1. Use trigonometric relationships to determine lengths and angle measures; i.e., Law of Sines and Law of Cosines.

11-12AIIGSB B. Represent transformations within a coordinate system using vectors and matrices.
11AIIGSI 3. Describe multiplication of a vector and a scalar graphically and algebraically, and apply to problem situations

***Grade Level Indicators:**
Transformation and Symmetry.

AIIGSI 3. Apply transformations and coordinates in solving problems.
AIIGSI 4. Use deductive reasoning.

*Additional indictors from past course of study.

Patterns, Functions and Algebra Standard
Students use patterns, relations and functions to model, represent and analyze problem situations that involve variable quantities. Students analyze, model and solve problems using various representations such as tables, graphs and equations.

The student will be able to meet any previous objectives of this standard, and, in addition ….

Mathematical Processes Benchmarks Gr. 11-12:

AIIMPB	A.	Construct algorithms for multi-step and non-routine problems.
AIIMPB	B.	Construct logical verifications or counter-examples to test conjectures and to justify or refute algorithms and solutions to problems.
AIIMPB	C.	Assess the adequacy and reliability of information available to solve a problem.
AIIMPB	D.	Select and use various types of reasoning and methods of proof.
AIIMPB	E.	Evaluate a mathematical argument and use reasoning and logic to judge its validity.
AIIMPB	F.	Present complete and convincing arguments and justifications, using inductive and deductive reasoning, adapted to be effective for various audiences.
AIIMPB	G.	Understand the difference between a statement that is verified by mathematical proof, such as a theorem, and one that is verified empirically using examples or data.
AIIMPB	H.	Use formal mathematical language and notation to represent ideas, to demonstrate relationships within and among representation systems, and to formulate generalizations.
AIIMPB	I.	Communicate mathematical ideas orally and in writing with a clear purpose and appropriate for a specific audience.
AIIMPB	J.	Apply mathematical modeling to workplace and consumer situations, including problem formulation, identification of a mathematical model, interpretation of solution within the model, and validation to original problem situation.

Essential Focus:
None found.

Benchmarks Gr. 8-10:
Indicators Gr. 10:

8-10AIIPFAB	A.	Generalize and explain patterns and sequences in order to find the next term and the *n*th term.		
		10AIIPFAI	2.	Describe and compare characteristics of the following families of functions: square root, cubic, absolute value and basic trigonometric functions; e.g., general shape, possible number of roots, domain and range.
8-10AIIPFAB	B.	Identify and classify functions as linear or nonlinear, and contrast their properties using tables, graphs or equations.		
		10AIIPFAI	5.	Solve simple linear and nonlinear equations and inequalities having square roots as coefficients and solutions.
8-10AIIPFAB	C.	Translate information from one representation (words, table, graph or equation) to another representation of a relation or function.		
		10AIIPFAI	12.	Describe the relationship between slope of a line through the origin and the tangent function of the angle created by the line and the positive *x*- axis.

8-10AIIPFAB D. Use algebraic representations, such as tables, graphs, expressions, functions and inequalities, to model and solve problem situations.

8-10AIIPFAB E. Analyze and compare functions and their graphs using attributes, such as rates of change, intercepts and zeros.
- 10AIIPFAI 4. Use algebraic representations and functions to describe and generalize geometric properties and relationships.
- 10AIIPFAI 9. Recognize and explain that the slopes of parallel lines are equal and the slopes of perpendicular lines are negative reciprocals.

8-10AIIPFAB F. Solve and graph linear equations and inequalities.
- 10AIIPFAI 6. Solve equations and inequalities having rational expressions as coefficients and solutions.
- 10AIIPFAI 7. Solve systems of linear inequalities.

8-10AIIPFAB G. Solve quadratic equations with real roots by graphing, formula and factoring.
- 10AIIPFAI 8. Graph the quadratic relationship that defines circles.
- 10AIIPFAI 10. Solve real-world problems that can be modeled using linear, quadratic, exponential or square root functions.

8-10AIIPFAB H. Solve systems of linear equations involving two variables graphically and symbolically.
- 10AIIPFAI 3. Solve equations and formulas for a specified variable; e.g., express the base of a triangle in terms of the area and height.
- 10AIIPFAI 11. Solve real-world problems that can be modeled, using systems of linear equations and inequalities.

8-10AIIPFAB I. Model and solve problem situations involving direct and inverse variation.

8-10AIIPFAB J. Describe and interpret rates of change from graphical and numerical data.

Benchmarks Gr. 11-12:
Indicators Gr. 11:

11-12AIIPFAB A. Analyze functions by investigating rates of change, intercepts, zeros, asymptotes, and local and global behavior. .
- 11AIIAPFAI 10. Describe the characteristics of the graphs of conic sections.
- 11AIIAPFAI 11. Describe how a change in the value of a constant in an exponential, logarithmic or radical equation affects the graph of the equation.

11-12AIIPFAB B. Use the quadratic equations that have complex roots.
- 11AIIPFAI 1. Identify and describe problem situations involving an iterative process that can be represented as a recursive function; e.g., compound interest.
- 11AIIPFAI 8. Solve equations involving radical expressions and complex roots.

11-12AIIPFAB C. Use recursive functions to modal and solve problems; e.g., home mortgagees, annuities.

11-12AIIPFAB D. Apply algebraic methods to represent and generalize problem situations involving vectors and matrices.
- 11AIIPFAI 7. Model and solve problems with matrices and vectors.

11AIIPFAI	9.	Solve 3 by 3 systems of linear equations by elimination and using technology, and interpret graphically what the solution means (a point, line, plane, or no solution).

***Grade Level Indicators:**
Benchmarks 10-12:

Use Patterns, Relations And Functions

AIIPFAI	1.	Analyze the behavior of arithmetic and geometric sequences and series as the number of terms increases.
AIIPFAI	2.	Translate between the numeric and symbolic form of a sequence or series.
AIIPFAI	3.	Set up and solve systems of equations using matrices and graphs, with and without technology.

Analyze Change

AIIPFAI	4.	Understand the connection between circular and trigonometric functions.
AIIPFAI	5.	Use circular functions to model periodic real-world phenomena.
AIIPFAI	6.	Use a graphing calculator and computer software to graph various functions and relations.
AIIPFAI	7.	Use coordinate plane arguments in making geometric proofs.
AIIPFAI	8.	Explore the geometric basis for the functions of trigonometry.
AIIPFAI	9.	Represent sequences and series as functions, both algebraically and graphically.
AIIPFAI	10.	Describe and solve algebraic situations with matrices.
AIIPFAI	11.	Describe and use the inverse relationship between functions including exponential and logarithmic.
AIIPFAI	12.	Analyze and describe the errors and the sources of error that can be made when using computers and calculators to solve problems.
AIIPFAI	13.	Decide when a problem situation is best solved using a computer, calculator, paper and pencil, or mental arithmetic/estimation techniques.
AIIPFAI	14.	Examine complex numbers as zeros of functions.
AIIPFAI	15.	Translate verbal statements into symbolic language.
AIIPFAI	16.	Simplify algebraic expressions.
AIIPFAI	17.	Use the laws of exponents (including scientific notation).

*Additional indicators from past course of study.

Data Analysis and Probability Standard:
Students pose questions and collect, organize, represent, interpret and analyze data to answer those questions. Students develop and evaluate inferences, predictions and arguments that are based on data.

The student will be able to meet any previous objective of this standard, and, in addition ….

Mathematical Processes Benchmarks Gr. 11-12:

AIIMPB	A.	Construct algorithms for multi-step and non-routine problems.
AIIMPB	B.	Construct logical verifications or counter-examples to test conjectures and to justify or refute algorithms and solutions to problems.
AIIMPB	C.	Assess the adequacy and reliability of information available to solve a problem.
AIIMPB	D.	Select and use various types of reasoning and methods of proof.
AIIMPB	E.	Evaluate a mathematical argument and use reasoning and logic to judge its validity.
AIIMPB	F.	Present complete and convincing arguments and justifications, using inductive and deductive reasoning, adapted to be effective for various audiences.
AIIMPB	G.	Understand the difference between a statement that is verified by mathematical proof, such as a theorem, and one that is verified empirically using examples or data.
AIIMPB	H.	Use formal mathematical language and notation to represent ideas, to demonstrate relationships within and among representation systems, and to formulate generalizations.
AIIMPB	I.	Communicate mathematical ideas orally and in writing with a clear purpose and appropriate for a specific audience.
AIIMPB	J.	Apply mathematical modeling to workplace and consumer situations, including problem formulation, identification of a mathematical model, interpretation of solution within the model, and validation to original problem situation.

Essential Focus:

1. Students should know how to find functions that model data.
2. Students should know how to create, implement, and interpret simulations to model a statistical study.

Benchmarks Gr. 8-10:
Indicators Gr. 9-10:

8-10AIIDPB	A.			Create, interpret and use graphical displays and statistical measures to describe data; e.g., box-and-whisker plots, histograms, scatterplots, measures of center and variability.
		9AIIDPI	2.	Create a scatterplot for a set of bivariate data, sketch the line of best fit, and interpret the slope of the line of best fit.
		9AIIDPI	3.	Analyze and interpret frequency distributions based on spread, symmetry, skewness, clusters and outliers.
		9AIIDPI	6.	Make inferences about relationships in bivariant data, and recognize the difference between evidence of relationship (correlation) and causation.
		10AIIDPI	2.	Represent and analyze bivariate data using appropriate graphical displays (scatterplots, parallel box-and-whisker plots, histograms with more than one set of data, tables, charts, spreadsheets) with and without technology.
		10AIIDPI	3.	Display bivariate data where at least one variable is categorical.
		10AIIDPI	4.	Identify outliers on a data display; e.g., use interquartile range to identify outliers on a box-and-whisker plot.

		10AIIDPI	6.	Interpret the relationship between two variables using multiple graphical displays and statistical measures; e.g., scatterplots, parallel box-and-whisker plots, and measures of center and spread.
8-10AIIDPB	B.	Evaluate different graphical representations of the same data to determine which is the most appropriate representation for an identified purpose.		
8-10AIIDPB	C.	Compare the characteristics of the mean, median and mode for a given set of data, and explain which measure of center best represents the data.		
8-10AIIDPB	D.	Find, use and interpret measures of center and spread, such as mean and quartiles, and use those measures to compare and draw conclusions about sets of data.		
		10AIIDPI	1.	Describe measures of center and the range verbally, graphically and algebraically.
8-10AIIDPB	E.	Evaluate the validity of claims and predictions that are based on data by examining the appropriateness of the data collection and analysis.		
8-10AIIDPB	F.	Construct convincing arguments based on analysis of data and interpretation of graphs.		
8-10AIIDPB	G.	Describe sampling methods and analyze the effects of method chosen on how well the resulting sample represents the population.		
		9AIIDPI	4.	Describe and compare various types of studies (survey, observation, experiment), and identify possible misuses of statistical data.
		9AIIDPI	5.	Describe characteristics and limitations of sampling methods, and analyze the effects of random versus biased sampling; e.g., determine and justify whether the sample is likely to be representative of the population.
		10AIIDPI	5.	Provide examples and explain how a statistic may or may not be an attribute of the entire population; e.g., intentional or unintentional bias may be present.
8-10AIIDPB	H.	Use counting techniques, such as permutations and combinations, to determine the total number of options and possible outcomes.		
		9 IIADPI	7.	Use counting techniques and the Fundamental Counting principle to determine the total number of possible outcomes for mathematical situations.
8-10AIIDPB	I.	Design an experiment to test a theoretical probability, and record and explain results.		
		9AIIDPI	8.	Describe, create and analyze a sample space and use it to calculate probability.
8-10AIIDPB	J.	Compute probabilities of compound events, independent events, and simple dependent events.		
		9AIIDPI	9.	Identify situations involving independent and dependent events, and explain differences between and common misconceptions about probabilities associated with those events.
		10AIIDPI	8.	Differentiate between the probability of an event and the odds of an event and explain the relationship between probability and odds.
8-10AIIDPB	K.	Make predictions based on theoretical probabilities and experimental results.		

9AIIDPI	10.	Use theoretical and experimental probability, including simulations or random numbers, to estimate probabilities and to solve problems dealing with uncertainty; e.g., compound events, independent events, simple dependent events.

Honors Algebra II with Trigonometry

The standards, benchmarks, and indicators for this course are the same as those found for Advanced Algebra in this document. The trigonometry benchmarks and indicators from Sttistics in this document will also apply to this course. With honors students, the following expectations are included for both teacher and student:

1. Go at a faster pace
2. Cover the entire text
3. Use explorations, extensions, and all related computer and graphing calculator materials.
4. Supplement the course content with extended but related topics that will vary according to teacher background and student ability.
5. Supplement content with topics that vary according to background and interest of both the instructor and the students.
6. Participate in selected math team competitions such as the Ohio Math League, OCTM Math Contest, and the AMC 10 National Math Contest.

Data Analysis and Probability Standard
Students pose questions and collect, organize, represent, interpret and analyze data to answer those questions. Students develop and evaluate inferences, predictions and arguments that are based on data**.**

Mathematical Processes Benchmarks Gr. 11-12:

STATMPB	A.	Construct algorithms for multi-step and non-routine problems.
STATMPB	B.	Construct logical verifications or counter-examples to test conjectures and to justify or refute algorithms and solutions to problems.
STATMPB	C.	Assess the adequacy and reliability of information available to solve a problem.
STATMPB	D.	Select and use various types of reasoning and methods of proof.
STATMPB	E.	Evaluate a mathematical argument and use reasoning and logic to judge its validity.
STATMPB	F.	Present complete and convincing arguments and justifications, using inductive and deductive reasoning, adapted to be effective for various audiences.
STATMPB	G.	Understand the difference between a statement that is verified by mathematical proof, such as a theorem, and one that is verified empirically using examples or data.
STATMPB	H.	Use formal mathematical language and notation to represent ideas, to demonstrate relationships within and among representation systems, and to formulate generalizations.
STATMPB	I.	Communicate mathematical ideas orally and in writing with a clear purpose and appropriate for a specific audience.
STATMPB	J.	Apply mathematical modeling to workplace and consumer situations, including problem formulation, identification of a mathematical model, interpretation of solution within the model and validation to original problem situation.

Essential Focus:

1. Students should know how to find functions that model data
2. Students should know how to create, implement, and interpret simulations to model a statistical study.

Benchmarks Gr. 8-10:
Indicators Gr. 9-10:

8-10AIIDPB	A.	Create, interpret and use graphical displays and statistical measures to describe data; e.g., box-and-whisker plots, histograms, scatterplots, measures of center and variability.		
8-10AIIDPB	B.	Evaluate different graphical representations of the same data to determine which is the most appropriate representation for an identified purpose.		
		9STATDPI	1.	Classify data as univariate (single variable) or bivariate (two variables) and as quantitative (measurement) or qualitative (categorical) data.
8-10AIIDPB	C.	Compare the characteristics of the mean, median and mode for a given set of data, and explain which measure of center best represents the data.		
		10STATDPI	7.	Model problems dealing with uncertainty with area models (geometric probability).

8-10AIIDPB D. Find, use and interpret measures of center and spread, such as mean and quartiles, and use those measures to compare and draw conclusions about sets of data.

8-10AIIDPB E. Evaluate the validity of claims and predictions that are based on data by examining the appropriateness of the data collection and analysis.

8-10AIIDPB F. Construct convincing arguments based on analysis of data and interpretation of graphs.

8-10AIIDPB G. Describe sampling methods and analyze the effects of method chosen on how well the resulting sample represents the population.

8-10AIIDPB H. Use counting techniques, such as permutations and combinations, to determine the total number of options and possible outcomes.

8-10AIIDPB I. Design an experiment to test a theoretical probability, and record and explain results.

8-10AIIDPB J. Compute probabilities of compound events, independent events, and simple dependent events.

8-10AIIDPB K. Make predictions based on theoretical probabilities and experimental results.

Benchmarks Gr. 11-12:
Indicators Gr. 11-12:

11-12STATDPB A. Create and analyze tabular and graphical displays of data using appropriate tools, including spreadsheets and graphing calculators.
- 11STATDPI 6. Create a scatterplot of bivariate data, identify trends, and find a function to model the data
- 12STATDPI 2. Transform bivariate data so it can be modeled by a function; e.g., use logarithms to allow nonlinear relationship to be modeled by linear function.
- 12STATDPI 4. Apply the concept of a random variable to generate and interpret probability distributions, including binomial, normal and uniform.

11-12STATDPB B. Use descriptive statistics to analyze and summarize data, including measures of center, dispersion, correlation and variability.
- 11STATDPI 3. Describe how a linear transformation of univariate data affects range, mean, mode and median.
- 11STATDPI 5. Use technology to find the Least Squares Regression Line, the regression coefficient, and the correlation coefficient for bivariate data with a linear trend, and interpret each of these statistics in the context of the problem situation.
- 11STATDPI 8. Analyze and interpret univariate and bivariate data to identify patterns, note trends, draw conclusions, and make predictions.
- 12STATDPI 3. Describe the shape and find all summary statistics for a set of univariate data, and describe how a linear transformation affects shape, center and spread.
- 11STATDPI 7. Describe the standard normal curve and its general properties, and answer questions dealing with data assumed to be normal.

11-12STATDPB C. Design and perform a statistical experiment, simulation or study; collect and interpret data; and use descriptive statistics to communicate and support predictions and conclusions.
- 10STATDPI 7. Model problems dealing with uncertainty with area models (geometric probability).
- 11STATDPI 1. Design a statistical experiment, survey or study for a problem; collect data for the problem; and interpret the data with appropriate graphical displays, descriptive statistics, concepts of variability, causation, correlation and standard deviation.

11STATDPI 6. Use technology to compute the standard deviation and interpret that statistic.
11STATDPI 7. Describe the standard normal curve and its general properties, and answer questions dealing with data assumed to be normal.
11STATDPI 9. Evaluate validity of results of a study based on characteristics of the study design, including sampling method, summary statistics and data analysis techniques.
12STATDPI 1. Identify and use various sampling methods (voluntary response, convenience sample, random sample, stratified random sample, census) in a study.
12STATDPI 5. Use sampling distributions as the basis for informal inference.

11-12STATDPB D. Connect statistical techniques to applications in workplace and consumer situations.
11STATDPI 1. Design a statistical experiment, survey or study for a problem; collect data for the problem; and interpret the data with appropriate graphical displays, descriptive statistics, concepts of variability, causation, correlation and standard deviation.
11STATDPI 2. Describe the role of randomization in a well-designed study, especially as compared to a convenience sample, and the generalization of results from each.
11STATDPI 9. Evaluate validity of results of a study based on characteristics of the study design, including sampling method, summary statistics and data analysis techniques.
11STATDPI 10. Understand and use the concept of random variable, and compute and interpret the expected value for a random variable in simple cases.
11STATDPI 11. Examine statements and decisions involving risk; e.g., insurance rates and medical decisions.
12STATDPI 6. Use theoretical and experimental probability, including simulations or random numbers, to estimate probabilities and to solve problems dealing with uncertainty; including probability with unbiased or compound events.

**Benchmarks

STATDPB • Conduct a simple probability experiment and draw conclusions about the likelihood of possible outcomes.

STATDPB • Use counting techniques, such as permutations and combinations, to determine the total number of options and possible outcomes.

STATDPB • Design an experiment to test a theoretical probability, and record and explain results.

STATDPB • Use experimental or theoretical probability, including simulations, to represent, interpret and evaluate the expected outcomes in real;-world situations involving conditional probabilities, disjoint events, and complementary events.

STATDPB • Compute probabilities for complementary, compound and conditional events.

STATDPB • Interpret the meaning of probability in frequently used contexts, such as odds and risk.

STATDPB • Make predictions based on theoretical probabilities and experimental results.

**Added Benchmarks

***Grade Level Indicators:**

Data Collection

STATDPI	1.	Describe measures of center and the range verbally, graphically and algebraically.
STATDPI	2.	Describe bivariate data using appropriate graphical displays (scatterplots, parallel box-and-whisker plots, histograms with more than one set of data, tables, charts, spreadsheets) with and without technology.
STATDPI	3.	Display bivariate data where at least one variable is categorical.
STATDPI	4.	Identify outliers on a data display; e.g., use interquartile range to identify outliers on a box-and-whisker plot.

Statistical Methods

STATDPI	5.	Analyze and interpret frequency distributions based on spread, symmetry, skewness, clusters and outliers.
STATDPI	6.	Describe and compare various types of studies (survey, observation, experiment), and identify possible misuses of statistical data.
STATDPI	7.	Describe characteristics and limitations of sampling methods, and analyze the effects f random versus biased sampling; e.g., determine and justify whether the sample is likely to be representative of the population.
STATDPI	8.	Make inferences about relationships in bivariant data, and recognize the difference between evidence f relationship (correlation) and causation.
STATDPI	9.	Provide examples and explain how a statistic may or may not be an attribute of the entire population.
STATDPI	10.	Interpret the relationship between two variables using multiple graphical displays and statistical measures; e.g., scatterplots, parallel box-and-whisker plots, and measures of center and spread.

Probability

STATDPI	11.	Use counting techniques and the fundamental counting principle to determine the total number of possible outcomes for mathematical situations.
STATDPI	12.	Describe, create and analyze a sample space and use it to calculate probability.
STATDPI	13.	Identify situations involving independent and dependent events, and explain differences between and common misconceptions about, probabilities associated with those events.
STATDPI	14.	Differentiate and explain the relationship between the probability of an event and the odds of an event, and compute one given the other.
STATDPI	15.	Use theoretical or experimental probability, including simulations, to determine probabilities in real-world problem situations involving uncertainty, such as mutually exclusive events, complementary events, and conditional probability.
STATDPI	16.	Compute probabilities for independent events; compound events, and mutually exclusive events, using theoretical probability experimental probability (simulations) as needed.

* Additional indicators from past course of study.

PRE-CALCULUS AND DISCRETE MATHEMATICS (PDM) Number , Number Sense and Operations Standard

Number, Number Sense Standard:
Students demonstrate number sense including an understanding of number systems and operations, and how they relate to one another. Students compute fluently and make reasonable estimates using paper and pencil, technology-supported and mental methods.

Mathematical Processes Benchmarks Gr. 11-12:

PDMMPB A. Construct algorithms for multi-step and non-routine problems.
PDMMPB B. Construct logical verifications or counter-examples to test conjectures and to justify or refute algorithms and solutions to problems.
PDMMPB C. Assess the adequacy and reliability of information available to solve a problem.
PDMMPB D. Select and use various types of reasoning and methods of proof.
PDMMPB E. Evaluate a mathematical argument and use reasoning and logic to judge its validity.
PDMMPB F. Present complete and convincing arguments and justifications, using inductive and deductive reasoning, adapted to be effective for various audiences.
PDMMPB G. Understand the difference between a statement that is verified by mathematical proof, such as a theorem, and one that is verified empirically using examples or data.
PDMMPB H. Use formal mathematical language and notation to represent ideas, to demonstrate relationships within and among representation systems, and to formulate generalizations.
PDMMPB I. Communicate mathematical ideas orally and in writing with a clear purpose and appropriate for a specific audience.
PDMMPB J. Apply mathematical modeling to workplace and consumer situations, including problem formulation, identification of a mathematical model, interpretation of solution within the model and validation to original problem situation.

Essential Focus:

1. Students should know how to use rational exponents.

Benchmarks Gr. 11-12:
Indicators Gr. 11-12:

11-12PDMNB A. Demonstrate that vectors and matrices are systems having some of the same properties of the real number system.

11-12PDMNB B. Develop an understanding of properties of and representations for addition and multiplication of vectors and matrices.

11-12PDMNB C. Apply factorials and exponents, including fractional exponents, to solve practical problems.
12PDMNI 2. Apply combinations as a method to create coefficients for the Binomial Theorem, and make connections to everyday and workplace problem situations.

11-12PDMNB D. Demonstrate fluency in operations with real numbers, vectors and matrices, using mental computation or paper and pencil calculations for simple cases, and technology for more complicated cases.

11-12PDMNB E. Represent and compute with complex numbers.

12PDMNI	1.	State which properties (closure, identity, inverse, commutative and associative) hold for operations with complex numbers.

***Grade Level Indicators:**

Number and Number Systems

PDMNI 1. Determine what properties (closure, identity, inverse, commutative and associative) hold for operations with complex numbers.

Computation and Estimation

PDMNI 2. Compute sums, differences, products and quotients of complex numbers.

PDMNI 3. Apply factorials and numbers with negative or fractional exponents to solve problems.

*Additional indictors from past course of study.

PRE-CALCULUS AND DISCRETE MATHEMATICS (PDM) Measurement Standard

Measurement Standard
Students estimate and measure to a required degree of accuracy and precision by selection and using appropriate units, tools and technologies.

Mathematical Processes Benchmarks Gr. 11-12:

PDMMPB	A.	Construct algorithms for multi-step and non-routine problems.
PDMMPB	B.	Construct logical verifications or counter-examples to test conjectures and to justify or refute algorithms and solutions to problems.
PDMMPB	C.	Assess the adequacy and reliability of information available to solve a problem.
PDMMPB	D.	Select and use various types of reasoning and methods of proof.
PDMMPB	E.	Evaluate a mathematical argument and use reasoning and logic to judge its validity.
PDMMPB	F.	Present complete and convincing arguments and justifications, using inductive and deductive reasoning, adapted to be effective for various audiences.
PDMMPB	G.	Understand the difference between a statement that is verified by mathematical proof, such as a theorem, and one that is verified empirically using examples or data.
PDMMPB	H.	Use formal mathematical language and notation to represent ideas, to demonstrate relationships within and among representation systems, and to formulate generalizations.
PDMMPB	I.	Communicate mathematical ideas orally and in writing with a clear purpose and appropriate for a specific audience.
PDMMPB	J.	Apply mathematical modeling to workplace and consumer situations, including problem formulation, identification of a mathematical model, interpretation of solution within the model and validation to original problem situation.

Essential Focus:

1. Students should know how to analyze error approximation.
2. Students should know how to use successive approximations to understand the concept of limit**.**
3. Students should know how to use unit (dimensional) analysis in measurement computations**.**

Benchmarks Gr. 11-12:
Indicators Gr. 11-12:

11-12PDMMB	A.	Explain differences among accuracy, precision and error, and describe how each of those can affect solutions in measurement situations.		
11-12PDMMB	B.	Apply various measurement scales to describe phenomena and solve problems.		
		11PDMMI	2.	Use radian and degree angle measures to solve problems and perform conversions as needed.
		11PDMMI	3.	Derive a formula for the surface area of a cone as a function of its slant height and the circumference of its base.
11-12PDMMB	C.	Estimate and compute areas and volume in increasingly complex problem situations.		

		11PDMMI	4.	Calculate distances, areas, surface areas and volumes of composite three-dimensional objects to a specified number of significant digits.
11-12PDMMB	D.	Solve problem situations involving derived measurements; e.g., density, acceleration.		
		11PDMMI	5.	Solve real-world problems involving area, surface area, volume and density to a specified degree of precision.
		12PDMMI	2.	Use radian measures in the solution of problems involving angular velocity and acceleration.

***Grade Level Indicators:**

PDMMI	1.	Solve problems involving derived measurements; e.g., acceleration and pressure.
PDMMI	2.	Apply informal concepts of successive approximation, upper and lower bounds, and limits in measurement situations; e.g., measurement of some quantities, such as volume of a cone, can be determined by sequences of increasingly accurate approximations.

*Additional indicators from past course of study.

Geometry and Spatial Sense Standard
Students identify, classify, compare and analyze characteristics, properties and relationships of one-, two- and three-dimensional geometric figures and objects. Students use spatial reasoning, properties of geometric objects, and transformations to analyze mathematical situations and solve problems.

Mathematical Processes Benchmarks Gr. 11-12:

PDMMPB A. Construct algorithms for multi-step and non-routine problems.
PDMMPB B. Construct logical verifications or counter-examples to test conjectures and to justify or refute algorithms and solutions to problems.
PDMMPB C. Assess the adequacy and reliability of information available to solve a problem.
PDMMPB D. Select and use various types of reasoning and methods of proof.
PDMMPB E. Evaluate a mathematical argument and use reasoning and logic to judge its validity.
PDMMPB F. Present complete and convincing arguments and justifications, using inductive and deductive reasoning, adapted to be effective for various audiences.
PDMMPB G. Understand the difference between a statement that is verified by mathematical proof, such as a theorem, and one that is verified empirically using examples or data.
PDMMPB H. Use formal mathematical language and notation to represent ideas, to demonstrate relationships within and among representation systems, and to formulate generalizations.
PDMMPB I. Communicate mathematical ideas orally and in writing with a clear purpose and appropriate for a specific audience.
PDMMPB J. Apply mathematical modeling to workplace and consumer situations, including problem formulation, identification of a mathematical model, interpretation of solution within the model and validation to original problem situation.

Essential Focus:

1. Students should know how to use trigonometric relationships to determine lengths and angle measures in any triangle.
2. Students should know how to visualize and represent objects in 3-space**.**
3. Students should know how to develop geometric relationships using the coordinate system
4. Students should know how to apply trigonometric relationships among the angle and sides of a right triangle.
5. Students should know how to establish the validity of geometric conjectures and critique arguments made by others.

Benchmarks Gr. 11-12:
Indicators Gr. 11-12:

11-12PDMGSB A. Use trigonometric relationships to verify and determine solutions in problem situations.
- 11PDMGSI 4. Use trigonometric relationships to determine lengths and angle measures; i.e., Law of Sines and Law of Cosines.
- 11PDMGSI 5. Identify, sketch and classify the cross sections of three-dimensional objects.
- 12PDMGSI 2. Derive and apply the basic trigonometric identities; i.e., angle addition, angle subtraction and double angle.

11-12PDMGSB B. Represent transformations within a coordinate system using vectors and matrices. Prove or disprove conjectures and solve problems involving two- and three-dimensional objects represented within a coordinate system.
- 11PDMGSI 1. Use polar coordinates to specify locations on a plane.

11PDMGSI 2. Represent translations using vectors.
12PDMGSI 1. Use matrices to represent translations, reflections, rotations, dilations and their compositions.

***Grade Level Indicators:**

Visualization and Geometric Models

PDMGSI 1. Relate graphical and algebraic representations of lines, simple curves and conic sections.
PDMGSI 2. Recognize and compare specific shapes in multiple geometries.
PDMGSI 3. Use discrete math models to make conjectures and solve problems.
PDMGSI 4. Use various representations to demonstrate the effects of multiple transformations.

*****Additional indictors from past course of study.

PRE-CALCULUS AND DISCRETE MATHEMATICS (PDM) Patterns, Functions, and Algebra Standard

Patterns, Functions and Algebra Standard
Students use patterns, relations and functions to model, represent and analyze problem situations that involve variable quantities. Students analyze, model and solve problems using various representations such as tables, graphs and equations**.**

Mathematical Processes standard Benchmarks Gr. 11-12:

PDMMPB	A.	Construct algorithms for multi-step and non-routine problems.
PDMMPB	B.	Construct logical verifications or counter-examples to test conjectures and to justify or refute algorithms and solutions to problems.
PDMMPB	C.	Assess the adequacy and reliability of information available to solve a problem.
PDMMPB	D.	Select and use various types of reasoning and methods of proof.
PDMMPB	E.	Evaluate a mathematical argument and use reasoning and logic to judge its validity.
PDMMPB	F.	Present complete and convincing arguments and justifications, using inductive and deductive reasoning, adapted to be effective for various audiences.
PDMMPB	G.	Understand the difference between a statement that is verified by mathematical proof, such as a theorem, and one that is verified empirically using examples or data.
PDMMPB	H.	Use formal mathematical language and notation to represent ideas, to demonstrate relationships within and among representation systems, and to formulate generalizations.
PDMMPB	I.	Communicate mathematical ideas orally and in writing with a clear purpose and appropriate for a specific audience.
PDMMPB	J.	Apply mathematical modeling to workplace and consumer situations, including problem formulation, identification of a mathematical model, interpretation of solution within the model and validation to original problem situation.

Essential Focus:

1. Students should know how to find functions that model data.
2. Students should know how to apply graphing concepts and function notation to nonlinear functions.
3. Student should know how logarithmic and exponential functions are related.
4. Students should know the properties and characteristics of polynomial and trigonometric functions.
5. Students should know how to create multiple symbolic representations of a function or relation that satisfies given conditions.

Benchmarks Gr. 11-12:
Indicators Gr. 11-12:

11-12PDMPFAB	A.			Analyze functions by investigating rates of change, intercepts, zeros, asymptotes, and local and global behavior.
		11PDMPFAI	3.	Describe and compare the characteristics of the following families of functions: quadratics with complex roots, polynomials of any degree, logarithms, and rational functions; e.g., general shape, number of roots, domain and range, asymptotic behavior.
		11PDMPFAI	4.	Identify the maximum and minimum points of polynomial, rational and trigonometric functions graphically and with technology.
		11PDMPFAI	5.	Identify families of functions with graphs that have rotation symmetry or reflection symmetry about the *y*-axis, *x*-axis or $y = x$.
		11PDMPFAI	10.	Describe the characteristics of the graphs of conic sections.

	11PDMPFAI	11.	Describe how a change in the value of a constant in an exponential, logarithmic or radical equation affects the graph of the equation.
	12PDMPFAI	9.	Translate freely between polar and Cartesian coordinate systems.

11-12PDMPFAB B. Use the quadratic formula to solve quadratic equations that have complex roots.

11-12PDMPFAB C. Use recursive functions to model and solve problems; e.g., home mortgages, annuities.

11-12PDMPFAB D. Apply algebraic methods to represent and generalize problem situations involving vectors and matrices.

***Grade Level Indicators**

PDMPFAI 1. Translate a recursive function into a closed form expression or formula for the *n*th term to solve a problem situation involving an iterative process; e.g., find the value of an annuity after 7 years.

PDMPFAI 2. Analyze the behavior of arithmetic and geometric sequences and series as the number of terms increases.

PDMPFAI 3. Translate between the numeric and symbolic form of a sequence or series.

PDMPFAI 4. Represent the inverse of a function symbolically and graphically as a reflection about $y = x$.

PDMPFAI 5. Use an open sentence (algebraic equation) to symbolize a problem situation and solve the equation to find a solution to the problem.

PDMPFAI 6. Validate and generalize problem situations.

PDMPFAI 7. Select appropriate notation and methods for symbolizing the problem statement and the solution process.

PDMPFAI 8. Use the following problem-solving strategies;

- Look for a pattern
- Make a graph
- Guess and check
- Make a drawing or diagram
- Make a model
- Construct a table
- Restate the problem
- Identify a sub goal
- Identify given, needed and extraneous information
- Account for all possibilities
- Write an open sentence
- Solve a simpler or similar problem
- Work backwards

PDMPFAI 9. Describe and compare the characteristics of transcendental and periodic functions; e.g., general shape, number of roots, domain and range, asymptotic behavior, extreme, local and global behavior.

Use Algebraic Representations

PDMPFAI 10. Represent the inverse of a transcendental function symbolically.

PDMPFAI 11. Model and solve problems with matrices and vectors.

PDMPFAI 12. Solve equations involving radical expressions and complex roots.

PDMPFAI 13. Solve 3 by 3 systems of linear equations by elimination and using technology, and interpret graphically what the solution means (a point, line, plane, or no solution).

PDMPFAI 14. Set up and solve systems of equations using matrices and graphs, with and without technology.

PDMPFAI 15. Make arguments about mathematical properties using mathematical induction.

PDMPFAI 16. Make mathematical arguments using the concepts of limit.

PDMPFAI 17. Compare estimates of the area under a curve over a bounded interval by partitioning the region with rectangles; e.g., make successive estimates using progressively smaller rectangles.

PDMPFAI 18. Recognize and explain that the slopes of parallel lines are equal and the slopes of perpendicular lines are negative reciprocals.

Analyze Change

PDMPFAI 19. Use the concept of limit to find instantaneous rate of change for a point on a graph as the slope of a tangent at a point.

PDMPFAI 20. Create multiple symbolic representations of a function or relation that satisfies given conditions.

PDMPFAI 21 Identify and represent the pattern of recursive functions.

PDMPFAI 23. Represent a function as a composite of two or more simpler functions.

PDMPFAI 24. Determine the appropriateness of parametric representations in applications.

PDMPFAI 25. Investigate and apply rates of change with respect to parameterization.

PDMPFAI 26. Determine the results of parameter changes on the graphs of trigonometric functions.

PDMPFAI 27. Distinguish between linear and non-linear rates of change.

PDMPFAI 28. Determine the class of function that most appropriately models a given condition.

PDMPFAI 29. Apply trigonometric identities to represent functions in equivalent forms.

PDMPFAI 30. Combine, compose, and invert functions to perform transformations.

PDMPFAI 31. Determine the properties and characteristics of polynomial, exponential, logarithmic, circular, power and linear functions.

PDMPFAI 32. Determine the most appropriate representation for a function.

*Additional indicators from past course of study.

Data Analysis and Probability Standard:
Students pose questions and collect, organize, represent, interpret and analyze data to answer those questions. Students develop and evaluate inferences, predictions and arguments that are based on data.

Mathematical Processes Benchmarks Gr. 11-12:

PDMMPB	A.	Construct algorithms for multi-step and non-routine problems.
PDMMPB	B.	Construct logical verifications or counter-examples to test conjectures and to justify or refute algorithms and solutions to problems.
PDMMPB	C.	Assess the adequacy and reliability of information available to solve a problem.
PDMMPB	D.	Select and use various types of reasoning and methods of proof.
PDMMPB	E.	Evaluate a mathematical argument and use reasoning and logic to judge its validity.
PDMMPB	F.	Present complete and convincing arguments and justifications, using inductive and deductive reasoning, adapted to be effective for various audiences.
PDMMPB	G.	Understand the difference between a statement that is verified by mathematical proof, such as a theorem, and one that is verified empirically using examples or data.
PDMMPB	H.	Use formal mathematical language and notation to represent ideas, to demonstrate relationships within and among representation systems, and to formulate generalizations.
PDMMPB	I.	Communicate mathematical ideas orally and in writing with a clear purpose and appropriate for a specific audience.
PDMMPB	J.	Apply mathematical modeling to workplace and consumer situations, including problem formulation, identification of a mathematical model, interpretation of solution within the model and validation to original problem situation.

Essential Focus:

1. Students should know how to find functions that model data.
2. Students should know how to create, implement and interpret simulations to model a statistical study.

Benchmarks Gr. 11-12:
Indicators Gr. 11-12:

11-12PDMDPB	A.	Create and analyze tabular and graphical displays of data using appropriate tools, including spreadsheets and graphing calculators.
11-12PDMDPB	B.	Use descriptive statistics to analyze and summarize data, including measures of center, dispersion, correlation and variability.
11-12PDMDPB	C.	Design and perform a statistical experiment, simulation or study; collect and interpret data; and use descriptive statistics to communicate and support predictions and conclusions.
11-12PDMDPB	D.	Connect statistical techniques to applications in workplace and consumer situations.

**** Benchmarks**

- PDMDPB • Conduct a simple probability experiment and draw conclusions about the likelihood of possible outcomes.
- PDMDPB • Use counting techniques, such as permutations and combinations, to determine the total number of options and possible outcomes.
- PDMDPB • Design an experiment to test a theoretical probability, and record and explain results.
- PDMDPB • Use experimental or theoretical probability, including simulations, to represent, interpret and evaluate the expected outcomes in real;-world situations involving conditional probabilities, disjoint events, and complementary events.
- PDMDPB • Compute probabilities for complementary, compound and conditional events.
- PDMDPB • Interpret the meaning of probability in frequently used contexts, such as odds and risk.
- PDMDPB • Make predictions based on theoretical probabilities and experimental results.

******Additional benchmarks from past course of study.

***Grade Level Indicators:**

- PDMDPI 1. Use an open sentence (algebraic equation to symbolize a problem situation and solve the equation to find a solution to the problem.
- PDMDPI 2. Validate and generalize problem situations.
- PDMDPI 3. Select appropriate notation and methods for symbolizing the problem statement and the solution process.
- PDMDPI 4. Use the following problem-solving strategies:
  - Look for a pattern
  - Make a graph
  - Guess and check
  - Make a drawing or diagram
  - Make a model
  - Construct a table
  - Restate the problem
  - Identify a subgoal
  - Identify given, needed and extraneous information
  - Account for all possibilities
  - Write an open sentence
  - Solve a simpler or similar problem
  - Work backwards

*Additional indicators from past course of study.

Honors Pre-Calculus

The standards, benchmarks, and indicators are the same as those found in this Math Course of Study for Pre-Calculus Discrete Math (PDM). With honors students, the following expectations are included for both teacher and student.

1. Go at a faster pace.
2. Cover the entire text.
3. Use explorations, extensions, and all related computer and graphing calculator materials.
4. Extend and apply the data analysis benchmarks and indictors from Statistics course of study.
5. Supplement the course content with extended and related topics that will vary according to background and interests of both the instructor and the students.
6. Participate in selected math team competitions such as the Ohio Math League, OCT Math Contest, AMC 12 National Math Contest.

Advanced Placement Calculus

Olmsted Falls High School participates in the Advanced Placement Calculus program in conjunction with the College Board. The AP program provides a carefully and thoroughly detailed syllabus for the AB course of Calculus. This syllabus is sufficient for the purpose of planning.

APPLIED MATHEMATICS WITH COMPUTER APPLICATIONS I AND II

INTRODUCTION:

These semester courses will explore mathematical topics through the use of the graphing calculator and the computer. The emphasis will be on learning mathematics. Some time will be devoted to learning computer and graphing calculator programming, however, so students can use technology to explore more fully the mathematics being learned.

These courses assume no prior experience with programming. If, in fact, students enrolled in the course have significant facility with programming, the programming component can be minimized and more time spent on mathematical units. For students with little or no prior experience with programming, at least half of the class time should be spent on computer and calculator programming. Other lessons on programming may be necessary throughout the year as students determine that more sophisticated programming techniques or commends are necessary to solve specific mathematical problems. However, this is a mathematics course concerned primarily with ideas, concepts, skills, and problems of various branches of mathematics. The computer and graphing calculator are tools used to study mathematics; the technology is not the primary object in the course.

The extent to which computers are used instead of graphing calculators is dependent upon both student and teacher background. The difficulty level of problems and sophistication of programming techniques are clearly a function of the audience. Various computer languages may be used to solve problems or present programming techniques. At present, BASIC, PASCAL, PC LOGO AND C++ are the computer languages that may be used in these semester courses. The teacher is the best judge in what course of action to follow as he prepares the course for his students. Students will have exposure to programming n the computer as well as the TI-83 and the TI – 92 graphing calculators sometime during each of these two semester classes. Much depends on the make-up of the student population.

The objectives that follow are by no means complete. They are meant as a guide for the teacher. Ultimately, the instructor must decide how he can formulate significant mathematical problems for students to solve using programming techniques. It is further assumed that the problems selection will remain in the spirit of the NCTM Standards.

Advanced Placement Computer Science

Olmsted Falls High School participates in the Advanced Placement Computer Science program in conjunction with the College Board. The AP program provides a carefully and thoroughly detailed syllabus for the AB course of Computer Science. This syllabus is sufficient for the purpose of planning.

SMART ESSENTIAL FOCUS

K – 2 Number and Number Sense
ESSENTIAL FOCUS:

1. Students should know how to count, order and compare objects and numbers. (K)
2. Students should know the concept of place value and the operations of addition and subtraction. (1st)
3. Students should know how to add and subtract fluently with single digit numbers. (2nd)
4. Students should know how to apply strategies for the addition and subtraction of two-digit numbers. (2nd)
5. Students should know it is possible to estimate quantities without knowing them exactly. (2nd)

K-2 Measurement
ESSENTIAL FOCUS:

1. Students should know how to recognize intervals of time on a calendar (k).
2. Students should know how to use nonstandard units to measure (1st).
3. Students should know how to use units of time (2nd).
4. Students should know how to use standard units of measure (2nd).

K-2 Geometry
ESSENTIAL FOCUS:

1. Students should know how to identify and describe common shapes (k).
2. Students should know how to identify and compare two-dimensional shapes (1st).
3. Students should know how to manipulate shapes to recognize congruency with transformations. (2nd).

K-2 Patterns, Functions and Algebra
ESSENTIAL FOCUS:

1. Students should know how to sort objects according to an attribute, (K).
2. Students should know how to recognize and extend patterns (1st).
3. Students should know how to generalize a pattern and determine a rule (2nd).

K-2 Data Analysis and Probability
ESSENTIAL FOCUS:

1. Students should know how to represent data by sorting and classifying objects according to attributes. (K)
2. Students should know how to collect, sort, and analyze data (1st).
3. Students should know how to read and interpret graphs and charts (2nd).
4. Students should know how to verify or justify their conclusions drawn from data (2nd).

3 – 4 Number and Number Sense
ESSENTIAL FOCUS:

1. Students should know how to add and subtract fluently with two- and three-digit numbers (3rd).
2. Students should know how to model, represent and explain multiplicative reasoning (3rd).

3. Students should know factions are parts of a whole (3rd).
4. Student should know how to multiply fluently (4th).
5. Students should know how to model, represent and explain division (4th)
6. Students should know when answers are reasonable. (4th).

3-4 Measurement
ESSENTIAL FOCUS:

1. Students should know tht standard units of measurements are necessary (3rd).
2. Students should know how to convert whole number measurements within each system (4th).
3. Students should know how to define and determine perimeter (4th).

3-4 Geometry
ESSENTIAL FOCUS:

1. Students should know how to identify, compare, and analyze attributes of two- and three-dimensional shapes (3rd).
2. Students should know how to use appropriate vocabulary to describe geometric properties and relationships (4th).

3-4 Probability, Functions and Algebra
ESSENTIAL FOCUS:

1. Students should know how to write a mathematical expression or equation using symbols and letters (3rd).
2. Students should know how to apply the distributive property (4th).

3-4 Data Analysis and Probability
ESSENTIAL FOCUS:

1. Students should know how to translate information among charts, tables, and graphs (3rd).
2. Students should know how to describe the likelihood of event (3rd).
3. Students should know how to appropriately represent and interpret data (4th).
4. Students should know how to relate basic probability concepts to realistic situations (4th).

5 – 7 Number and Number Sense
ESSENTIAL FOCUS:

1. Students should know how to compare and order fractions, decimals and percents (5/6th).
2. Students should know how to convert among fractions, decimals and percents (5/6th).
3. Students should know how to compute fluently with fractions and decimals (5/6th).
4. Students should know how to compute with integers and percents (7th).

5-7 Measurement
ESSENTIAL FOCUS:

1. Students should know how to select appropriate units to measure perimeter, area, and volume (6th).
2. Students should know how to find area and perimeter using formulas. (6th).
3. Students should know how to use proportional reasoning to make conversions within each measurement system. (7th).

5-7 Geometry
ESSENTIAL FOCUS:

1. Students should know how to describe sizes, positions, and orientations of shapes after rotations, dilations, reflections and translations. (6th)
2. Students should know how to describe and classify fundamental relationships among groups or three-dimensional shapes (7th).
3. Students should know how to identify and plot points on a rectangular coordinate plane in all our quadrants (7^{th}).

5-7 Patterns, Functions and Algebra
ESSENTIAL FOCUS:

1. Students should know how to validate a given solution for an equation or inequality (5^{th})
2. Students should know how to apply formulas in problem solving situations. (6^{th}).
3. Students should know how to relate and compare different forms of representations for a pattern (7^{th}).
4. Students should know that the value of a variable can change (7^{th}).

5-7 Data Analysis and Probability
ESSENTIAL FOCUS:

1. Students should know how to formulate appropriate questions to investigate relevant issues (6^{th}).
2. Students should know how to find, interpret, and appropriately use measures of central tendency and spread (6th).
3. Students should know the sum of the probabilities of all possible outcomes is one (6^{th}).
4. Students should know how to distinguish between theoretical and experimental probability (7^{th}).

8-10 Number and Number Sense
ESSENTIAL FOCUS:

1. Students should know how to use scientific notation and exponents to represent large and small numbers (8^{th}).
2. Students should know how to solve problems using proportional relationships (8^{th}).
3. Students should know how to estimate, compute, and solve problems that include rational numbers (8^{th}).
4. Students should know how to use matrices (9^{th}).
5. Students should know how to compute fluently with real numbers including powers and roots (9^{th}).
6. Students should know how to represent vectors (10^{th}).

8-10 Measurement
ESSENTIAL FOCUS:

1. Students should know how to use algebraic formulas to determine an unknown measure. (8^{th}).
2. Students should know how to use unit (dimensional) analysis in measurement computations (9th).
3. Students should know how to distinguish between precision and accuracy in measurement situation (10^{th}).

8-10 Geometry
ESSENTIAL FOCUS:

1. Students should know how to apply geometric relationships using the coordinate system (9^{th}) .
2. Students should know how to apply trigonometric relationships among the angles and sides of a right triangle (9^{th}).
3. Students should know how to establish the validity of geometric conjectures using deduction (10^{th}).
4. Students should know how to analyze transformations in the rectangular coordinate system (10^{th}).

8-10 Patterns, Functions and Algebra
ESSENTIAL FOCUS:

1. Students should know how to analyze relationships between linear equations and their graphs, connecting the meaning of intercepts and slope to the context of the situation (8^{th}).
2. Students should know to recognize and generate equivalent forms of algebraic expressions and equations with justifications (8^{th}).
3. Students should know how to apply graphing concepts and function notation to non-linear functions (9^{th}).
4. Students should know how to logarithmic and exponential functions are related (10^{th}).

8-10 Data Analysis and Probability
ESSENTIAL FOCUS:

1. Students should know how to find a line of best fit and use it to make appropriate predictions (8^{th}).
2. Students should know how to use fundamental counting principles to determine probabilities of independent and simple dependent events (9^{th}).
3. Students should know how to use technology to find an appropriate equation of best fit to mode the data (10^{th}).
4. Students should know how to compute probabilities of compound events (10^{th}).

11-12 Numbers and Number Sense
ESSENTIAL FOCUS:

1. Students should know how to use rational exponents (11^{th}).

11-12 Measurement
ESSENTIAL FOCUS:

1. Students should know how to analyze error approximation. (11^{th}).
2. Students should know how to use successive approximations to understand the concept of limit (12^{th}).

11-12 Geometry
ESSENTIAL FOCUS:

1. Students should know how to use trigonometric relationships to determine lengths and angle measures in any triangle. (11^{th}).
2. Students should know how to visualize and represent objects in three-space (12^{th}).

11-12 Patterns, Functions and Algebra
ESSENTIAL FOCUS:

1. Students should know the properties and characteristics of polynomial and trigonometric functions (11^{th}).
2. Students should know how to create multiple symbolic representations of a function or relation that satisfied given conditions (12^{th}).

11-12 Data Analysis and Probability
ESSENTIAL FOCUS:

1. Students should know how to find functions that model data (11^{th}).
2. Students should know how to create, implement and interpret simulations to model a statistical study (12^{th}).